A. Losev: 2D analogs of 4D SUSY gauge theories 10/26

$\dim_{\mathbb{C}} = 0$ Quantum mech	$\dim_{\mathbb{C}} = 1$ σ-model	$\dim_{\mathbb{C}} = 2$ gauge theory
Toric varieties	Maps($\mathbb{P}^1 \to$ Toric)	Bundles on $\mathbb{P}^2$
Complete intersections	Maps($\mathbb{P}^1 \to$ c.i)	
Grassmannians	Maps ($\mathbb{P}^1 \to$ Grassmann)	

ADHM: bundles on $\mathbb{P}^2$ given by eqn / nonabelian group: analog to hypersurfaces in Grassmannians

Simplest example of Nikita's story: in case of Maps ($\mathbb{C} \to \mathbb{P}^n$) get Mirror to Givental's approach to mirror symmetry

Consider space of quasimaps $QMaps(\mathbb{C} \to \mathbb{P}^n)$
2 groups acting: $\mathbb{C}^* \circlearrowright \mathbb{P}^1$, $(\mathbb{C}^*)^n \circlearrowright \mathbb{P}^n$

Nikita: bundles on $\mathbb{C}^2$ with triv:
$\mathbb{C}^* \times \mathbb{C}^* \circlearrowright \mathbb{C}^2$, $(\mathbb{C}^*)^{rk} \circlearrowright$ trivialization

Instantons $\longleftrightarrow$ holomorphic maps.
Rotation of triv $\longleftrightarrow$ rotation of target space
Sum over c_2 $\longleftrightarrow$ sum over degrees

Partition function $Z(a_1, \dots a_n; \varepsilon, \Lambda) =$ (simplify a bit)
almost equivariant volume of quasimaps space

$=$

Modify deRham differential using U(1) action:
$D(\varphi) = d + \varphi i_v$. $D_\varphi^2 = \varphi \mathcal{L}_v$.
Squares to zero on equivariant forms.

When locus of fixed pts is compact can make sense of equivariant integral of 1.

Cohomology of $\mathbb{CP}^n$ = $\oint \frac{P_1(\varphi) \dots P_n(\varphi)}{\varphi^{n+1}} d\varphi = \int_{\mathbb{CP}^n} \omega_1 \wedge \dots \omega_n$

... writing $\mathbb{CP}^n = \mathbb{C}^{n+1}/\mathbb{C}^*$, denominator comes from equivariant cohomology of $\mathbb{C}^{n+1}$...

Equivariance: ring of functions on fixed points of vector fields. Set equivariant parameters to zero => nilpotent cohomology ring.

equivalently $\oint \frac{P_1(\varphi) \dots P_n(\varphi)\, d\varphi}{(\varphi+a_0)(\varphi+a_1)\dots(\varphi+a_n)}$

Space of quasi-maps $\mathcal{O}(\mathbb{C}^*)^{n+1}$. Consider equivariant volume. Fixed point set is smooth → can write formula

$Z_{\varepsilon_1,\varepsilon_2} = \sum \Lambda^n \dots$ | $e^{\frac{1}{\varepsilon_1\varepsilon_2}F + \dots}$ $\mathcal{L} = \mathcal{L}_0 + F$

SYM, 6 monopoles | Seiberg-Witten: SYM, T above, add term to Lagrangian

$\updownarrow$ | $\updownarrow$

nonlinear σ-model, type A ($\mathbb{P}^1 \to \mathbb{P}^N, \dots$) curved | linear σ-model, w/ Landau-Ginzburg term $\mathcal{L} = \mathcal{L}_0 + V$

$Z = \sum \Lambda^d \dots$ | $e^{\frac{1}{\varepsilon}W}$ W LG potential.

Theories are not equivalent, but have same observables (BRST-cohomology)

Defs of superpotential in LG model = cohomology ring of $\mathbb{P}^N$

as space

A A_∞-algebra : $A^{\otimes n} \to A$ all n. $A = H^*(\mathbb{P}^n)$

Ginzburg comments

A-model : potential gives (when Taylor expanded) all these maps

B-model : instead of giving algebra give its spectrum :

... Spec (A_∞ algebra) is germ of A_∞ algebra.

Frob manifold : comm assoc multiplication on tangent bdle

- can think of as $\{T_X \to \mathrm{Hom}(T_X, T_X)\} = \underset{\phi}{\Omega^1 \otimes} \mathrm{End}(T_X)$

Frobenius property : this is a Higgs field $\phi \wedge \phi = 0$

Another POV : $\mathrm{Sym}\, T_X \to T_X$ from multiplication map.

so T_X is quotient of $\mathrm{Sym}\, T_X = \mathbb{C}[T_X^*]$

- so think of space as Lagrangian subspace of T^*. integrable system comes from quantizing this Lagrangian

Our space is formal completion of $H^*(\mathbb{P}^n)$

Taylor expansion of ϕ