Andrei Losev – T-duality etc. part 2

Note Title 1/12/2007

Topics
- T-duality in $d=2$ theory
- S-duality in $d=4$ theories as a generalization
  - operators (Wilson & 't Hooft)
  - branes

<u>T-duality</u> equivalence of two 2d theories

I. $\int \mathcal{D}\varphi \, e^{-R^2\int_\Sigma d\varphi \wedge * d\varphi}$ $\quad \Sigma \longrightarrow S^1$ radius R

II. $\int \mathcal{D}\varphi_D \, e^{-\frac{1}{R^2}\int_\Sigma d\varphi_D \wedge * d\varphi_D}$ $\quad \Sigma \longrightarrow S^1$ radius $1/R$

Not equivalent as classical theories: actions are different ... T-duality inverts the Planck constants.

Different geometries : S^1 radius R
$\longleftrightarrow$ S^1 radius $1/R$

Checking the equivalence: must compare correlators (the observables in the theory):
need map on local observables
[for simplicity put $R=1$]

basic identity $\quad d\varphi_D = i * d\varphi$

--- field φ is example of a local observable.

Puzzle! $d^2 = 0 \implies i\, d * d\varphi \overset{?}{=} 0$

(basic assumption: Σ is not changing ----. worldsheets stay same, so d on one side goes to d on other side)

but this is just the equation of motion.....

Consider the quadratic integral of exponential observable

$$\int \mathcal{D}\varphi \; e^{-\int d\varphi * d\varphi} \cdot \underbrace{e^{i\varphi(x)}}_{\text{tachyon observable}}$$

Now consider $\int \mathcal{D}\varphi \; e^{-\int d\varphi * d\varphi + i\varphi(x)} \cdot \underline{\underline{d\varphi(y)}} = \langle e^{i\varphi(x)} d\varphi(y) \rangle$

local observable

$= i\, d \log |x-y|^2$ 2d Green's function

But $d * d \log |x-y|^2 = \delta^{(2)}(x-y)$ 2-dim δ fn

so it's "probable" that $i\, d * d\varphi = 0$ —

but problem at source $x = y$.

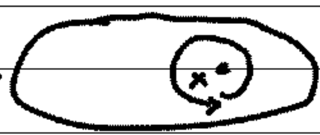
Consider integral $\int_{S^1} d\varphi_D$ & insert into correlators
around x

$$\left\langle e^{i\varphi(x)} \int_{S^1} d\varphi_D(y) \right\rangle = 1 \qquad \text{(up to } 2\pi\text{)}$$

[$d\varphi$ not closed, $\delta^2 = \int_{\dots}$]

So from φ_D point of view, φ_D itself is not at all defined at x .... not single valued at x, it wraps around x.

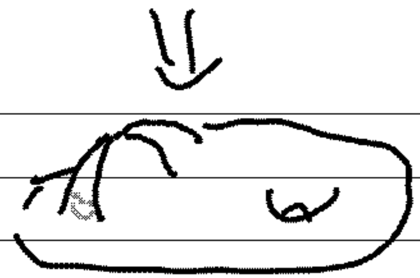
So we have a <u>vortex</u> at x:

Σ  $\xrightarrow{\varphi_D} S^1$ map which wraps nontrivially near x

Last time we obtained $d\varphi = 0$ by integrating over φ ..... doesn't apply at point of insertion of operator, we've created a singularity there.

Should we include vortices?
want to understand local observables
as a result of degenerations of partition functions

Take Σ  $\longrightarrow S^1$ & pinch handles of Σ

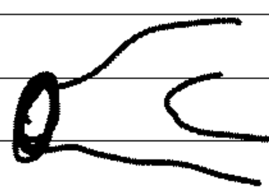


Σ $\longrightarrow S^1$

Want to be able to factorize (cut up Σ) & have some local observables

... in degenerated surface need to allow local observables like vortices to capture the limit of expectations around the vanished cycle.

T-duality interchanges the evaluation operator $\langle e^{i\varphi} \rangle$ with vortex operator.



So the space of boundary conditions is decomposed according to mapping classes to S^1

states = fns on boundary conditions

$$= \bigoplus_n \text{winding states } \tilde{W}_n$$

Evaluation operators preserve this decomposition:

insert local evaluation operator don't change winding #

On the other hand vortices change winding numbers!

$n_1 - n_2 = k$ winding numbers must be compatible

$\Rightarrow$ k-vortex adds k to winding #.

The number dual to winding number is the electric charge / "momentum of string" ....

<u>d=4</u> Electric-magnetic duality

A 1-form B 2-form

2D: $$\int_{\Sigma_2} i p \wedge d\varphi - \frac{1}{2R^2} p \wedge * p \rightsquigarrow$$

4D: $$\int_{\Sigma_4} i B \wedge dA - \frac{1}{2\left(\frac{1}{g_0^2}\right)} B \wedge * B$$

Can evaluate this integral in two ways!

- first take Gaussian integral over B

$$\Longrightarrow -\frac{1}{2g_0^2}\int_{\Sigma_4} dA \wedge * dA$$ quantum Maxwell theory

- first integrate over A

$$\Rightarrow \quad -\frac{1}{2\left(\frac{1}{g_0^2}\right)} \int_{\Sigma_4} dA_D \wedge * dA_D$$

Here $B = dA_D$.

Same warning as before: integral over A is valid <u>outside</u> points of insertion of local observable $A(x)$ ....

g_0^2 = electric charge

$\frac{1}{g_0^2} = g_D^2$ = charge of dual theory

Before we had evaluation observables: functions $e^{i\varphi(x)}$ could be evaluated at a point

Now we have 1-forms, which we can evaluate along a line $\rightsquigarrow$ holonomy $e^{i\int_\ell A}$

- so need also <u>line</u> operators, in addition to point operators & boundaries

Now let's enjoy the duality!

$dA = i * dA_D$

On a line A is not defined...

$F_D = dA_D$ field strength

$\int_{S^2} F_D = \int_{S^2} dA_D =$ linking number between S^2 & l

$\longrightarrow$ Such situation is called a monopole:
dual to electric field has flux along l

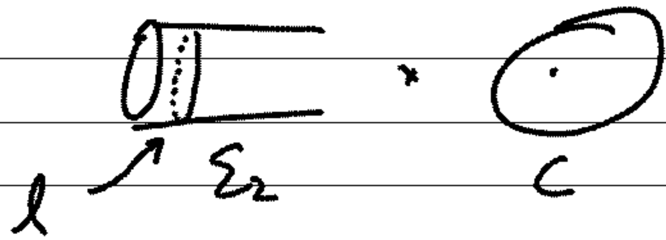
'tHooft line $e^{i\int_l A_D}$ in A_D picture,

or topological defect in A picture

....

Kapustin Witten. Now consider theory on $\Sigma_4 = \Sigma_2 \times C$ with $\partial\Sigma_2 \neq \phi$.

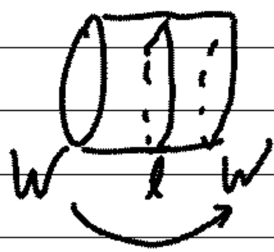
Now look at Wilson lines or 't Hooft lines



Line observables act on boundary conditions

Boundary conditions: fix gauge field on boundary + topological (winding #) conditions

W = fns on boundary conditions



U like operators

Can now classify <u>eigen</u> boundary conditions, & they will be interchanged by duality

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2d case: what are eigenvectors of operator $e^{i\varphi(x)}$ in W - fns on boundary conditions:

$$\varphi|_{\Gamma} = \varphi_0 \quad \text{constant}$$ (δ-function on point $\varphi_0 \in$ all bdry conditions)

$$\lim_{x \to x_0 \in \partial\Sigma} e^{i\varphi(x)} |W\varphi_0\rangle = e^{i\varphi_0} |W\varphi_0\rangle$$

Magnetic local observables shift winding #s

$\Longrightarrow$ to get eigenstates need sums over all these winding #s. (analog of Hecke operators shifting topological type)