(1) Let $\mathcal{E}(0) \coloneqq \Sigma^{-1}\widetilde{H}^*(\mathbb{RP}^2; \mathbb{F}_2)$.
   (a) Compute $\mathrm{Ext}^{*,*}_{\mathcal{A}(1)}(\mathcal{E}(0), \mathbb{F}_2)$ as a module over $\mathrm{Ext}(\mathbb{F}_2, \mathbb{F}_2)$. (Hint: try fitting $\mathcal{E}(0)$ into two different extensions to determine the entire module structure.)
   (b) Run the Adams spectral sequence and determine $\widetilde{ko}_*(\mathbb{RP}^2)$ for $* < 8$.

(2) Show $\Omega_3^{\mathrm{Spin}}(B\mathbb{Z}/2) \cong \mathbb{Z}/8$.

(3) Recall from yesterday that if $W$ denotes the Wu manifold, $H^*(W; \mathbb{F}_2) \cong \mathbb{F}_2[z_2, z_3]/(z_2^2, z_3^2)$, with $w_2 = z_2$ and $w_3 = z_3$. Through the Wu formula, this means $\mathrm{Sq}^1 z_2 = z_3$ and $\mathrm{Sq}^2 z_3 = z_2 z_3$.
   (a) Compute $\mathrm{Ext}^{*,*}_{\mathcal{A}(1)}(\widetilde{H}^*(W; \mathbb{F}_2), \mathbb{F}_2)$ as a module over $\mathrm{Ext}^{*,*}_{\mathcal{A}(1)}(\mathbb{F}_2, \mathbb{F}_2)$. Hint: use the long exact sequence technique.
   (b) Compute $\widetilde{ko}_*(W)$ in degrees 7 and below.
   (c) Challenge question: what is $\mathrm{Ext}_{\mathcal{A}}(\widetilde{H}^*(W; \mathbb{F}_2), \mathbb{F}_2)$? Feed this question to a computer program to determine the Ext groups and determine as many stable homotopy groups of $W$ as you can.

(4) Let $T$ be the Thom spectrum of the virtual bundle $2\sigma - 2 \to \mathbb{RP}^\infty$, where $\sigma \to \mathbb{RP}^\infty$ is the tautological line bundle. Exhibit a 2-local equivalence

$$ko \wedge T \simeq \bigoplus_{i=0}^{\infty} \Sigma^{4i} H\mathbb{Z}.$$

   (Answer: see http://www.rrb.wayne.edu/papers/osac2.pdf.)

(5) Challenge question: if $n = 2^\ell$ for $\ell \geq 2$, show that $ko_3(BC_n) \cong \mathbb{Z}/2 \oplus \mathbb{Z}/2n$. (Hint: with Adams on its own, it's not clear how to determine the differentials you need. For Atiyah-Hirzebruch on its own, it's not clear how to resolve the extension problem.)

(6) Challenge question: figure out the details of the Adams spectral sequence over $\mathcal{E}(1)$ (for $ku$-theory). Determine the $\mathcal{E}(1)$-module structure on $\widetilde{H}^*(\mathbb{RP}^2; \mathbb{Z}/2)$, and compute $ku_*(\mathbb{RP}^\infty)$ as a $ku_*$-module.