

PRELIMINARY EXAMINATION IN ANALYSIS
Part II, Complex Analysis
August 15, 2011

1. Use residues to prove that

$$\int_0^{\infty} \frac{\sqrt{x}}{x^4 + 1} dx = \frac{\pi \cos(\pi/8)}{2 + \sqrt{2}}.$$

2. Give an explicit description of all conformal self-maps of $\Omega = \mathbb{C} \setminus \{0\}$. Do the same for $\Omega = \mathbb{C} \setminus \{z_1, z_2, \dots, z_n\}$.
3. Let $n \mapsto f_n$ be a sequence of one-to-one analytic maps from the upper half plane $H = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$ into itself. If this sequence converges pointwise on H to a non-constant function f , show that f is one-to-one and analytic on H .
4. Determine the partial fractions expansion for $\frac{1}{e^z - 1}$.