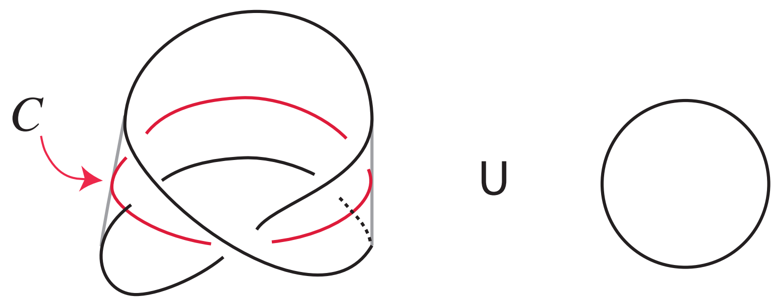
Preliminary Examination in Topology: January 2012

Algebraic Topology

Instructions: Do all three questions.

Time Limit: 90 minutes

For problems 1 and 2, consider the space K created by taking two copies of RP^2 and identifying them along the center curve of a mobius band in each. For clarification, think of $K_1 = K_2 = RP^2$ as obtained from the disjoint union of a mobius band and a disk



by identifying the boundary of the mobius band with the boundary of the disk. Then our complex K is made by identifying the curve C in K_1 with the curve C in K_2.

1.

(a). Write down the homology groups for RP^2 (no justification is necessary for this part). You may use these facts in doing part (b).

(b). Use the Mayer-Vietoris exact sequence to compute each of the zeroth, first, second, and third homology groups of K, thinking of K as $K_1 \cup K_2$.

2.

(a). Compute the fundamental group of K using the Seifert-Van Kampen Theorem. Simplify your presentation of the fundamental group as much as possible (i.e. with the fewest generators).

(b). Does K retract to the curve C? If so, describe the retraction. If not, prove there is no retraction.

(c). Describe the universal cover of K.

3. Let J, L be the two simple closed curves in the solid torus T, all contained in R^3 as shown below. Notice that the curve J bounds a taco-shaped disk in R^3 that misses L. Prove that J is not null-homotopic in the complement of L in T (i.e. with a homotopy missing L that lies entirely inside T). (Hint: Consider the universal cover of T).