

PRELIMINARY EXAMINATION IN ANALYSIS

Part II, Complex Analysis

January 9, 2013

1. Let f be a non-constant entire function on \mathbb{C} . Prove that f is a polynomial if and only if $\lim_{|z| \rightarrow \infty} |f(z)| = \infty$.
2. Let $\Omega \subset \mathbb{C}$ be open and simply connected. If $f : \Omega \rightarrow \Omega$ is analytic and has two fixed points, show that either $\Omega = \mathbb{C}$, or else f is the identity map on Ω .
3. Determine the partial fraction expansion for $\frac{1}{\sqrt{z} \sin \sqrt{z}}$.
4. Evaluate $\int_0^{\infty} \frac{\sin(x)}{x(x^2 + 1)} dx$.