Preliminary Examination in Topology: January 2013

Algebraic Topology portion

Instructions: Do all three questions.

Time Limit: 90 minutes.

1. Let X be the space obtained by attaching two 2-cells to $S^1 \times I$ by attaching maps f_0 and f_1 given by

$$f_0(z) = (z^2, 0)$$

$$f_1(z) = (z^3, 1)$$

where $z \in S^1$ is thought of as a unit complex number.

a. Compute $\pi_1(X)$.

b. Compute the homology groups $H_q(X)$.

2. What is the smallest g such that the closed orientable surface of genus g is a covering space of both the connected sum of 5 copies of P^2 and 5 copies of T^2?

3a. Define the *degree* of a map $f : S^n \to S^n$.

b. Let $f : S^{2n} \to S^{2n}$ be a map. Show that there exists $x \in S^{2n}$ such that either $f(x) = x$ or $f(x) = -x$. (Hint: you may use the fact that the antipodal map $S^n \to S^n$ has degree $(-1)^{n+1}$.)