PRELIMINARY EXAM IN ANALYSIS
PART II – COMPLEX ANALYSIS.

AUGUST 26, 2013 – TIME: 10:40–12:10.

Name (**print**): ______________________________________ UT ID: ____________

Please write clearly, and staple your work with the signed exam sheet on top !

Problem 1

Let Ω_1 and Ω_2 be two disjoint open subsets of the complex plane. Let $n \mapsto f_n$ be a sequence of analytic functions on Ω_1 , with values in Ω_2 . If this sequence converges pointwise to a function $f : \Omega_1 \to \mathbb{C}$, show that f is analytic and $f(\Omega_1) \subset \Omega_2$.

Problem 2

Assume that f is meromorphic on $\mathbb{C}$ satisfying $f(z) = f(-\frac{1}{\bar{z}})$ for all $z \in \mathbb{C}$, and $\lim_{z\to\infty} f(z) = 1$. Let $\gamma_\theta : \mathbb{R} \to \mathbb{C}$, $t \mapsto e^{i\theta}t$ be a contour that traces out a straight line at inclination angle θ containing the origin, and which contains no zeros or poles of f. Determine

$$\int_{\gamma_\theta} \frac{f'(z)}{f(z)} dz \,.$$

Problem 3

Determine all bijective conformal self-maps of $\Omega = \{z \in \mathbb{C} : |z| > 0\}$.

Problem 4

If $\Omega \subset \mathbb{C}$ is the complement of a compact connected set containing $\pm 1 \pm i$, show that there exists an analytic function $g : \Omega \to \mathbb{C}$ such that $g(z)^4 = z^4 + 4$.