The University of Texas at Austin
Department of Mathematics:e

The Preliminary Examination in Probability
Part II

Aug 19, 2013

Problem 1. Let $\{B_t\}_{t\in[0,\infty)}$ be the Brownian motion, and let σ be the last visit of the level 0 before time $t = 1$, i.e.,

$$\sigma = \sup\{t \leq 1 \, : \, B_t = 0\}.$$

(1) Show that σ is not a stopping time (*Hint:* Use the martingale $B_t^2 - t$.)
(2) Show that

$$\sigma \overset{(d)}{=} \tfrac{X^2}{X^2+Y^2},$$

where X, Y are independent unit normals.

Problem 2. Let M and N be two continuous local martingales. Show that $\mathcal{E}(M)\mathcal{E}(N)$ is a local martingale if and only if MN is a local martingale. (*Note:* $\mathcal{E}(X)_t = \exp(X_t - \frac{1}{2}[X,X]_t)$.)

Problem 3. Let $\{B_t\}_{t\in[0,\infty)}$ be the Brownian motion. For $c \in \mathbb{R}$ compute

$$\mathbb{P}[B_t + ct < 1, \text{ for all } t \geq 0\,].$$

(*Hint:* Either use Girsanov's theorem, of construct a function f such that $f(B_t + ct)$ is a martingale.)