Preliminary exam: Numerical Analysis, Part A, January 13, 2016

Name _____ , EID _____

1. A system of linear equations $Ax = b$ is given.

(a) Show that the system is nonsingular if A is strictly diagonally dominant.
(b) Define the Jacobi iterative method for the system and show that it will converge if A is strictly diagonally dominant.
(c) If the system is over determined, give the normal equation and pseudo invers forms for the least squares solution. Are they equivalent?

2. (a) Define Newton's method for minimization of a function $f(x), x \subset R^d$ which has a unique minimum at x_0.
(b) Prove the method converges if $d = 1$, $f(x) \subset C^3(R), f''(x_0) > 0$ and the initial value is close to the minimum.
(c) If the minimization is constrained such that $x \in \Omega \subset R^d$ describe a penalty method for the minimization and show convergence of your method if $d = 1$ and $f' > 0$.

3. Consider piecewise polynomial interpolation with N equidistant points $x_j = x_0 + jh, j = 1,2,..,N-1$ when the data $\{f(x_j)\}_{j=0}^{N-1}$ comes from a C^∞ function f.
($N = np + 1$, p: polynomial degree, n: number of intervals)
(a) Show that the interpolation error converges to 0 for fixed p as the distance between the points $h \to 0$.
(b) Prove the formula for the interpolation error.
(c) Give an error estimate in the H^1 Sobolev norm for $p = 1$ and prove you result.