1. Let $K$ be a smoothly embedded circle in $\mathbb{R}^3$ (i.e. a knot). Use a Mayer-Vietoris sequence to calculate the homology groups of $X = \mathbb{R}^3 \setminus K$ with $\mathbb{Z}$ coefficients. (The result does not depend on $K$).

2. Let $X = S^1 \vee S^1$ be the wedge of two circles. Recall that $\pi_1(X) = \langle a, b \rangle$ is free, generated by two loops, one going around each circle.
   (a): Draw a picture of a three-fold covering space $p : \tilde{X} \to X$ which is not normal. Write down generators for the corresponding subgroup $H$ of $\pi_1(X)$. What is the size of a minimal generating set for $H$?
   (b): Draw a picture of a three-fold covering space $q : \tilde{Y} \to X$ which is normal. Describe the action of the deck transformation group $G(\tilde{Y})$ in your picture.

3. Let $X$ be the identification space obtained from a cube $[0,1] \times [0,1] \times [0,1]$ by identifying opposite faces in pairs with a quarter twist in the positive direction. Let $Y$ denote the image of $[0,1] \times [0,1] \times \{0\}$ in $X$.
   (a): Write down presentations for $\pi_1(X, x_0)$ and $\pi_1(Y, x_0)$, where $x_0$ is one of the vertices. Do not simplify.
   (b): Compute the homology groups $H_1(X, \mathbb{Z})$ and $H_1(Y, \mathbb{Z})$.
   (c): Does $X$ retract onto $Y$?

4. Let $M_g$ denote the closed orientable surface of genus $g$. For what values of $n$ does there exist a connected normal covering space of $M_g$ whose deck transformation group is isomorphic to $\mathbb{Z}^n$?