1. Questions

(1) Classify all finitely generated abelian groups such that the group of automorphisms is finite.
   (b) What is \( \text{Aut}(\mathbb{Z} \times C_p \times C_q) \), for \( p \) and \( q \) relatively prime?

(2) (a) Prove that any group of order \( p^2 \) is abelian.
    (b) Classify the groups of order \( p^2 \).
    (c) Classify the non-abelian groups of order 8.

(3) Suppose that \( R \) is a commutative ring with an ideal \( I \subset R \) such that \( I^2 = I \).
    Show that \( R \cong I \oplus \tilde{R} \), where \( \tilde{R} \) is another ideal.

(4) Let \( G \) be a finite group and \( H \) a subgroup of finite index. Suppose that for any \( x, y \in G \setminus H \) we have \( xH \cap Hy \neq \emptyset \). What can you say about the order of \( G \)? (Hint: let \( G \) act on \( G/H \times G/H \).)

(5) Let \( R \) be a ring.
    (a) Show that a finitely presented \( R \)-module \( M \) is finitely generated. Recall that a module \( M \) is finitely presented if there exists an exact sequence
        \[ R^n \to R^m \to M \to 0. \]
    (b) Show that given a short exact sequence of \( R \)-modules
        \[ 0 \to P \to Q \to M \to 0 \]
        such that \( M \) is finitely presented and \( Q \) is finitely generated, then \( P \) is finitely generated.
    (c) Assume that \( R \) is Noetherian. Show that a finitely generated \( R \)-module \( M \) is finitely presented.
    (d) Give an example of a ring \( R \) and a module \( M \) which is finitely generated but not finitely presented.