Tangle Dynamics in a Toy Model of a Vortex **Bubble**

J.D. Mireles James

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J.D. Mireles James [3D Tangles](#page-113-0)

Introduction

Outline

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- [The Quadratic Family](#page-9-0)
- [Dynamics of the quadratic family](#page-18-0)
- [Computing Stable/Unstable Manifolds](#page-40-0)
- [Vortex Bubble Geometry](#page-87-0)
- [Tangle Dynamics in VPHM](#page-101-0)

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Talk Outline

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- \triangleright Introduce a simple family of quadratic volume-preserving discrete dynamical system.
- Introduce the notion of a "vortex-bubble".
- Study homoclinic tangle dynamics in the bubble of the quadratic family.
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The Map

The Volume Preserving Hénon Map, is given by

$$
f(x,y,z)=\left(\begin{array}{c}z+Q_{\alpha,\tau,a,b,c}(x,y)\\x\\y\end{array}\right),
$$

where *Q* is the quadratic function

$$
Q_{\alpha,\tau,a,b,c}(x,y) = \alpha + \tau x + a x^2 + b x y + c y^2,
$$

and $a + b + c = 1$.

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- \triangleright The Volume-Preserving Hénon map (VPHM) is a normal form for the family of quadratic diffeomorphisms with quadratic inverse.
- \triangleright The VPHM generalizes the classical area preserving Hénon family given by:

$$
f(x,y) = \left(\begin{array}{c}y + (1 - ax^2) \\ x\end{array}\right).
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Birds

Fixed Points

If $\tau^2 > 4\alpha$ there are exactly two fixed points $p_{0,1}$ located at

$$
p_{0,1} = \left(\begin{array}{c} x_{0,1} \\ x_{0,1} \\ x_{0,1} \end{array}\right),
$$

where

$$
x_{0,1} = \frac{-\tau}{2} \pm \frac{\sqrt{\tau^2 - 4\alpha}}{2}.
$$

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- \triangleright We refer to the first case as stability type $(2, 1)$, and the second case as stability type (1, 2).
- \triangleright On a large set of parameters, the VPHM has one fixed point p_0 of stability type (2, 1) and a second fixed point p_1 of stability type (1, 2).
- \blacktriangleright Then $W^u(p_0)$ and $W^s(p_1)$ are two dimensional manifolds which may intersect in such a way as to enclose a volume in phase space (Resonance Zone).
- \blacktriangleright In [DM09] it is shown that, on a parameter set of large measure, an elliptic invariant circle exists in the resonance zone. We refer to this situation a "vortex bubble".

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Regular/Integrable Vortex Bubble

Irregular Vortex Bubble [DM09]

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Some Bubbles in Physics: Hill's Vortex (flow)

Pekert and Sadlo (2007)

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Some Bubbles in Physics: Tokamak (PDE+Data)

Hayashi, Nizuguchi, &o (2001)

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Some Bubbles in Physics: Draft Tube (flow)

Pekert, Sado (Preprint)

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Some Bubbles in Physics: Natural Vortices

Mark Humpage and Atoniou, Lambropoulou

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- \triangleright The VPHM has been proposed as a toy model for 'vortex bubble' dynamics.
- \triangleright [DM09] presents a qualitative numerical study of elliptic dynamics inside the vortex-bubble of the VPHM (invariant circles, invariant tori, and their bifurcations).
- \triangleright We are interested in studying the boundary of the bubble itself, as well as homoclinic tangle dynamics near/in the bubble.

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Parameterization Method

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- \triangleright "The Parameterization Method for Invariant Manifolds II: Manifolds Associated to Non-Resonant Subspaces", Cabre, Fontich, de la Llave, I. U. of Math J. 52, 2003.
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Stable Manifolds of fixed Points

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Stable Manifolds of fixed Points

- \blacktriangleright Let $f: \mathbb{R}^n \to \mathbb{R}^n$ be a real analytic mapping with real analytic inverse.
- \triangleright Suppose that $f(p) = p$, and that $Df(p)$ is invertible and diagnalizable.
- \blacktriangleright Suppose that λ_1^s , ..., λ_k^s are *k* distinct stable eigenvalues of *Df*(p), ($|\lambda_i^s|$ < 1). Suppose that all the other eigenvalues of *Df*(*p*) have norm greater than or equal to one.
- \blacktriangleright Let Λ be the $k \times k$ matrix with diagonal entries λ_i^s .
- \blacktriangleright Let ξ_1^s, \ldots, ξ_k^s be eigenvectors associated with the stable eigenvalues and let *A* be the $n \times k$ matrix having ξ_i^s as columns.

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Parameterization for Stable Manifolds

$$
P(0) = p \tDP(0) = A
$$

as well as

$$
f[P(\theta)] = P[AB] \quad \theta \in B_r(0)
$$
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Parameterization for Stable Manifolds

 \triangleright The parameterization method consists of looking for an analytic mapping $P: B_r({\sf 0}) \subset \mathbb{R}^k \to \mathbb{R}^n$ satisfying;

$$
P(0) = p \quad DP(0) = A
$$

$$
f[P(\theta)] = P[\Lambda \theta] \quad \theta \in B_r(0) \tag{1}
$$

 $\left\{ \bigoplus_k k \right| \in \mathbb{R}^n, k \in \mathbb{N} \right\}$

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Schematic of the Parameterization Method

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Coefficient Computation

$$
P(\theta) = \sum_{|\alpha| \ge 0} a_{\alpha} \theta^{\alpha}, \quad a \in \mathbb{R}^n,
$$

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Coefficient Computation

▶ Since *P* solves a (functional) initial value problem with analytic data, assume that

$$
P(\theta) = \sum_{|\alpha| \geq 0} a_{\alpha} \theta^{\alpha}, \quad a \in \mathbb{R}^n,
$$

$$
\blacktriangleright a_{(0,\ldots,0)}=p,
$$

$$
\blacktriangleright \; a_{(0,\dots,1,\dots,0)} = \xi_i^s
$$

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Coefficient Computation

► Since P solves a (functional) initial value problem with analytic data, assume that

$$
P(\theta) = \sum_{|\alpha| \geq 0} a_{\alpha} \theta^{\alpha}, \quad a \in \mathbb{R}^n,
$$

multi-index notation; so $\theta \in \mathbb{R}^k$, $\alpha \in \mathbb{N}^k$, $|\alpha| = \alpha_1 + \ldots + \alpha_k, \theta^{\alpha} = \theta_1^{\alpha_1} \cdot \ldots \cdot \theta_k^{\alpha_k}, \mathbf{a}_{\alpha} \in \mathbb{R}^n.$ \blacktriangleright $a_{(0,...,0)} = p$, \blacktriangleright **a**_(0,...,1,...,0) = ξ_j^s .

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Coefficient Computation

► Since P solves a (functional) initial value problem with analytic data, assume that

$$
P(\theta) = \sum_{|\alpha| \geq 0} a_{\alpha} \theta^{\alpha}, \quad a \in \mathbb{R}^n,
$$

multi-index notation; so $\theta \in \mathbb{R}^k$, $\alpha \in \mathbb{N}^k$, $|\alpha| = \alpha_1 + \ldots + \alpha_k$, $\theta^{\alpha} = \theta_1^{\alpha_1} \cdot \ldots \cdot \theta_k^{\alpha_k}$, $a_{\alpha} \in \mathbb{R}^n$. \blacktriangleright $a_{(0,...,0)} = p$, \blacktriangleright **a**_(0,...,1,...,0) = ξ_j^s .

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Then substitute the unknown powerseries into Equation [\(1\)](#page-52-0), expand the nonlinearity, and match like powers of theta to work out the coefficients of the formal series:

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Coefficient Computation

Example the one-dimensional stable or unstable manifolds of p_0 in the VPHM has:

$$
\left(\begin{array}{ccc} \tau+2aa_0+bb_0-\lambda^n & ba_0+2cb_0 & 1\\ 1 & -\lambda^n & 0\\ 0 & 1 & -\lambda^n\end{array}\right)\left[\begin{array}{c} a_n\\ b_n\\ c_n\end{array}\right]=\left[\begin{array}{c} s_n\\ 0\\ 0\end{array}\right],
$$

where

$$
s_n = -\sum_{j=1}^{n-1} \left[a a_j a_{n-j} + b a_j b_{n-j} + c b_j b_{n-j} \right].
$$

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Coefficient Computation

One can check that the matrix equation has the form

 $[Df(p) - \lambda^n]$ $v_n = y_n$.

where *yⁿ* is a known quantity depending recursively on terms of order less than *n*.

The matrix equation can be solved uniquely as long as $\lambda^n \neq \lambda$. This condition holds for all $n > 2$ as $|\lambda| < 1$, so we can solve the system recursively for all the coefficients of order two and higher.

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Example: $W^u_{loc}(p_0)$ and $W^s_{loc}(p_0)$

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J.D. Mireles James [3D Tangles](#page-0-0)

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Globalization of the Stable and Unstable Manifolds :

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- \triangleright The unstable manifold is similar (it is the stable manifold of *f* −1).
- \triangleright Convergence of the series is treated in the references.
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The equations determining the coefficients of the two dimensional manifolds for the VPHM, as well as numerical implementation of the parameterization computations, are in:

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- [Computing Stable/Unstable Manifolds](#page-40-0)
- [Vortex Bubble Geometry](#page-87-0)
- [Tangle Dynamics in VPHM](#page-101-0)

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Set of Bounded Orbits [DM09]

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Set of Bounded Orbits [DM09]

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Vortex Bubbles Bifurcations: (horizontal)

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Tangle Dynamics in VPHM

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Bubble Tangle: homoclinic orbit

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Bubble Tangle: "Hairpin"

3D Tangle Geometry

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3D Tangle Geometry

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