Optimal and Suboptimal Routing Based on Partial CSI in MIMO Ad-Hoc Networks

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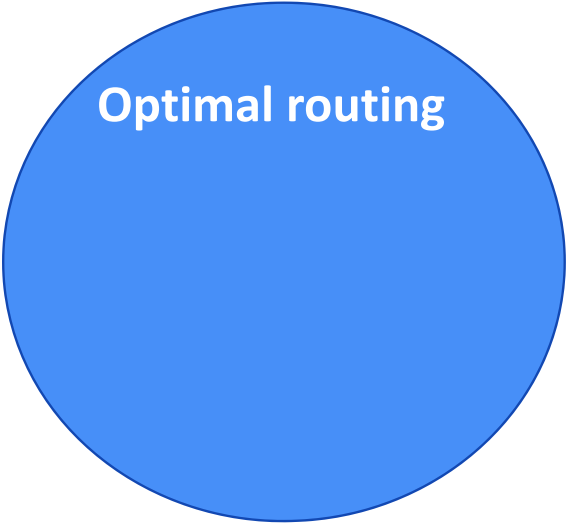
Outline

- Introduction & System Model.
- Optimal routing in random ad-hoc networks.
- Single antenna routing.
- MIMO routing.

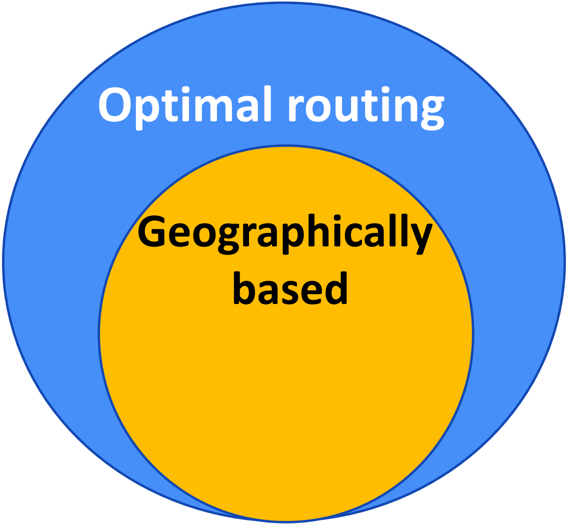
Statistical optimal routing

Routing in Wireless Ad Hoc Network



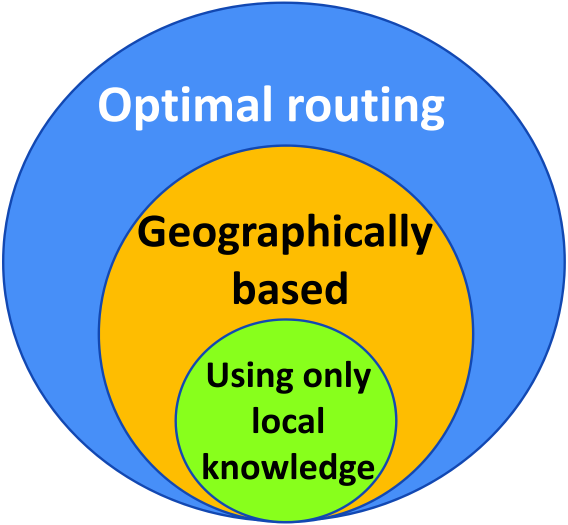
Statistical optimal routing

Routing in Wireless Ad Hoc Network



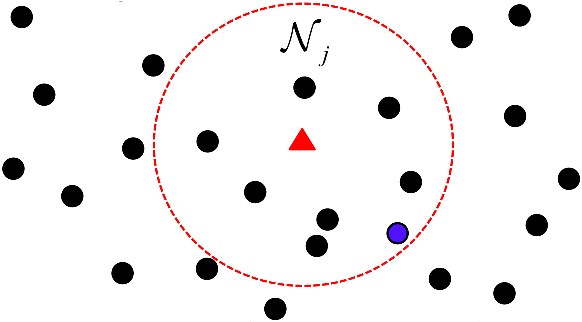
Statistical optimal routing

Routing in Wireless Ad Hoc Network



System model

- Slotted ALOHA MAC (p_{tx}).
- PPP distributed nodes (λ).
- Single / Multiple antennas.
- Local knowledge on nodes in routing zone.



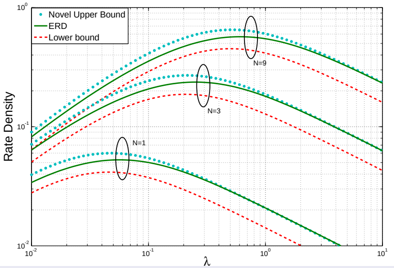
$$\mathcal{M}_j \triangleq \left\{ (\mathbf{r}_i, \mathbf{H}_{i,j}) : \forall i \in \mathcal{N}_j \right\}$$

Ergodic Rate Density (ERD)

$$\mathrm{R}(\lambda) = \lambda p_{tx} \cdot \mathbb{E}\left\{\log_2\left(1 + \mathsf{SIR}\right)\right\}$$

- Achievable upper bound on WANETs performance.
- Convenient for analysis.
- Good bounds.
- High complexity, large delay.

George et al, 2013 + 2015



Asymptotic density of rate and progress (ADORP)

$$\bar{D}\left(f(\cdot)\right) \triangleq \lambda p_{tx}\mathbb{E}\left\{ r_{f(\mathcal{M})} \log_2\left(1 + \mathrm{SIR}_{f(\mathcal{M})}\right) \right\}$$

- WANETs performance is measured by Rate×Progress.
- $f(\cdot)$ is the routing function.
- Use opportunistic relaying.
- No delay constraints.
- Implicit mobility.

Optimal routing

- In the single antenna case

$$\bar{D}\left(f(\cdot)\right) = \lambda p_{tx} \mathbb{E}\left\{ r_{f(\mathcal{M})} \log_2 \left(1 + \frac{S_{f(\mathcal{M})}}{J_{f(\mathcal{M})}} \right) \right\}$$

- Using the Law of Total Expectation

$$\bar{D}\left(f(\cdot)\right) = \lambda p_{tx} \mathbb{E}_{\mathcal{M}}\left\{ \mathbb{E}_{J|\mathcal{M}}\left\{ r_{f(\mathcal{M})} \log_2 \left(1 + \frac{S_{f(\mathcal{M})}}{J_{f(\mathcal{M})}} \right) \middle| \mathcal{M} \right\} \right\}$$

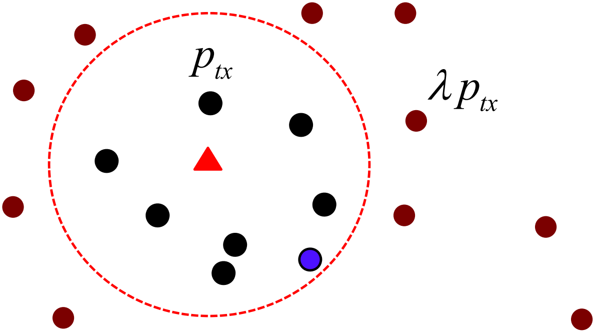
- SO: Statistical Optimal routing function

$$f_{\mathrm{SO}}(\mathcal{M}) \quad = \operatorname*{argmax}_{i \in \mathcal{N}} r_i \cdot \mathbb{E}\left\{ \log_2 \left(1 + \frac{S_i}{J_i} \right) \middle| \mathcal{M} \right\}$$

Depends on the distribution of $J_i|\mathcal{M}$.

Evaluation of the SO metric

- Uses only local knowledge.
- Takes into account interference statistics.
- Can be evaluated using Monte Carlo Simulations.
- Different statistics of nodes inside/outside the routing zone.



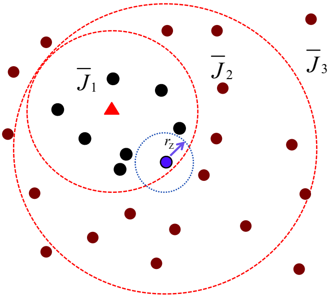
BO routing

- Lower bound on ERD

$$\mathbb{E}\left\{ \log_2\left(1+\frac{S_i}{J_i}\right) \middle| \mathcal{M} \right\} \geq p_{\mathrm{Z}}(i,\mathcal{M}) \log_2\left(1+\frac{S_i}{\mathbb{E}\{J_i | r > r_{\mathrm{Z}}, \mathcal{M}\}}\right)$$

- BO routing function

$$f_{\mathrm{BO}}(\mathcal{M}) = \operatorname*{argmax}_{i \in \mathcal{N}} p_{\mathrm{Z}}(i,\mathcal{M}) r_i \log_2\left(1+\frac{S_i}{\bar{J}_1^i + \bar{J}_2^i + \bar{J}_3^i}\right)$$



NSO routing

- **Narrow** knowledge

$$\mathcal{M}^i = \{\mathbf{r}_i, h_i\}$$

- Narrow Statistically Optimal routing function

$$f_{\text{NSO}}(\mathcal{M}) = \underset{i \in \mathcal{N}}{\text{argmax}}\, r_i \cdot \mathbb{E}\left\{\log_2\left(1 + \frac{S_i}{J_i}\right) \Big| \mathcal{M}^i\right\}$$

- Without knowledge on neighbors, the distribution of J_i is identical for **all** nodes.
- Interference distribution can be measured locally.
- Complexity is still quite high.

NBO routing

- Lower bound on ERD

$$p_{\mathrm{Z}}(i,\mathcal{M})\log_2\left(1+\frac{S_i}{\mathbb{E}\{J_i|r_{\min,i}>r_{\mathrm{Z}},\mathcal{M}^i\}}\right)$$
$$=\mathrm{const}\cdot\log_2(1+\gamma S_i)$$

- NBO routing function

$$f_{\mathrm{NBO}}(\mathcal{M})=\operatorname*{argmax}_{i\in\mathcal{N}} r_i\log_2(1+\gamma\cdot S_i)$$

where

$$\gamma=\frac{\alpha}{2}\left(\frac{\alpha-2}{\alpha\pi\lambda p_{tx}}\right)^{\frac{\alpha}{2}}$$

- Considers network parameters.
- Low complexity!

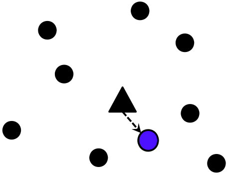
Numerical Results

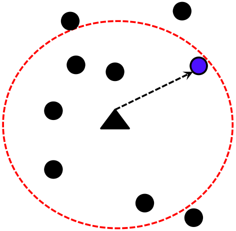
- Previously published geographic routing schemes
  - NiC: Nearest in a cone

$$f_{\text{Nearest}}(\mathcal{M}) = \operatorname*{argmin}_{i} r_i$$

  - MPR: Most progress within radius

$$f_{\text{MPR}}(\mathcal{M}) = \operatorname*{argmax}_{i:|r_{i,j}| \leq r_{\max}} r_i$$





Need to optimize the routing parameter (MPR)!

ADORP vs p_{tx}

Optimal and Suboptimal Routing

Introduction

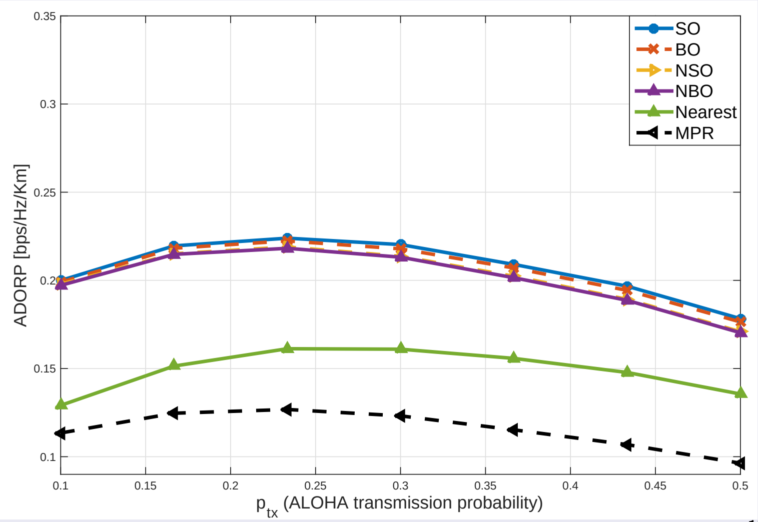
System Model

Single Antenna

Multiple Antennas

Conclusions

Single antenna, $\alpha = 3$



ADORP vs p_{tx}

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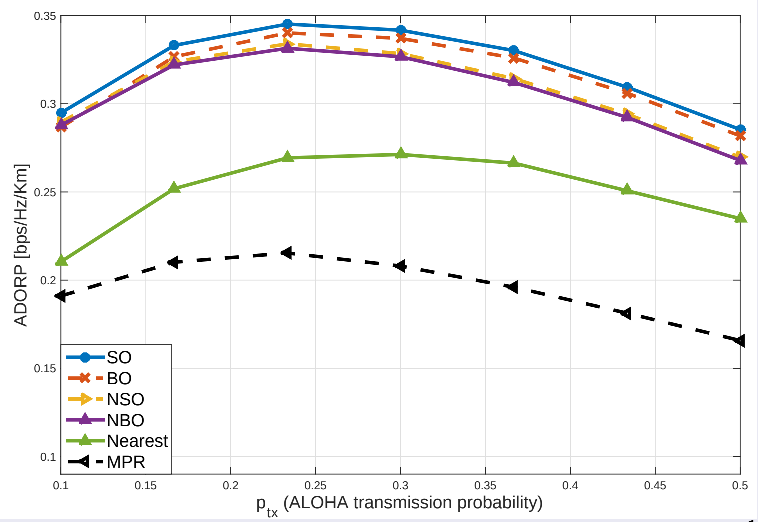
Single antenna, $\alpha = 4$



Multiple antennas WANET

- Optimal MIMO routing needs to select
  - Destination.
  - Number of streams.
  - Precoding vectors.
- Challenges:
  - Need to take into account the processing at the receiver.
  - The number of streams affects the distribution of the interference.
- Simplifying assumptions:
  - Tx: eigenbeamforming (equal power streams).
  - Rx: Partial ZF (of N_{ZF} nearest transmitters).
  - Consider only narrow knowledge.

Multiple antennas WANET

- ADORP

$$\bar{D}_o\left(f(\cdot)\right) = \lambda p_{tx} \mathbb{E}_{\mathcal{M}} \left\{ B\Big(f(\mathcal{M}), \mathcal{M}, K(\mathcal{M}) \Big) \right\}$$

Define

$$B(i, \mathcal{M}, K) \triangleq \mathbb{E}\left\{ r_i \sum_{k=1}^{K} \log_2 \left(1 + \frac{\frac{1}{K} r_i^{-\alpha} W_{i,k}}{J_{i,k}} \right) \middle| \mathcal{M}^i \right\}.$$

- $W_{i,k}$: Effective channel gain at the k-th stream
  - **Partially** known at the transmitter.

NSO routing

- Using narrow knowledge

$$f_{\mathrm{NSO}}(\mathcal{M}), K(\mathcal{M}) = \operatorname*{argmax}_{i,K} r_i \sum_{k=1}^{K} \mathbb{E}\left\{ \log_2 \left(1 + \frac{\frac{1}{K} r_i^{-\alpha} \gamma_{i,k}^2 Y_{i,k}}{J_{i,k}}\right) \middle| \mathcal{M}^i \right\}$$

- The distribution of J depends on the distribution of $K(\mathcal{M})$.
- High complexity.
- Intractable.

FSO routing

- Fixed number of streams per user

$$f_{\mathrm{FSO}}(\mathcal{M},K)=\operatorname*{argmax}_{i}\, r_i\sum_{k=1}^{K}\mathbb{E}\left\{\log_2\left(1+\frac{\frac{1}{K}r_i^{-\alpha}\gamma_{i,k}^2 Y_{i,k}}{J_{i,k}}\right)\middle|\mathcal{M}^i\right\}$$

- The distribution of $J_{i,k}$ is identical for any i, k.

LC routing function

- Taking the expectation of the numerator and the denominator

$$f_{\mathrm{LC}}(\mathcal{M}), K(\mathcal{M}) = \underset{i,K}{\operatorname{argmax}}\, r_i \cdot \sum_{k=1}^{K} \log_2\left(1 + \frac{\frac{1}{K} r_i^{-\alpha} \gamma_{i,k}^2 \bar{Y}}{\sigma_n^2 + C_{\alpha,N_{\mathrm{ZF}}}}\right)$$

where

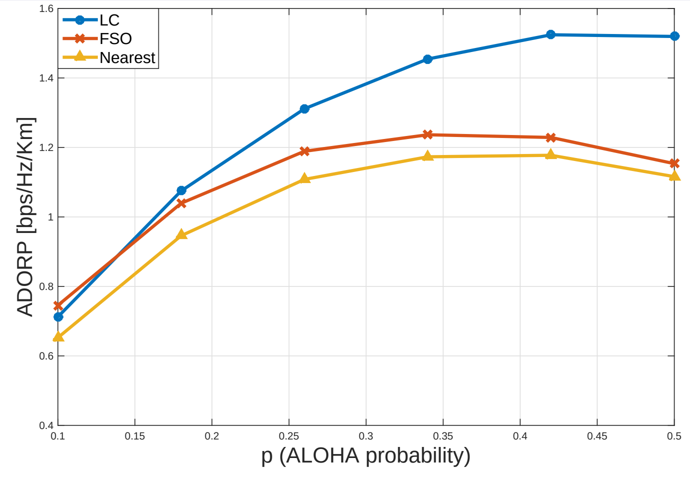
$$\bar{Y} \triangleq \frac{N_{\mathrm{R}} + 1 - K - \bar{T}_{\mathrm{ZF}}}{N_{\mathrm{R}} + 1 - K}$$

$\gamma_{i,k}^2$ is the k-th singular value, and $\bar{T}_{\mathrm{ZF}}$ is the average number of zeroed streams, and

$$C_{\alpha,N_{\mathrm{ZF}}} \triangleq \frac{2(\lambda p_{tx}\pi)^{\frac{\alpha}{2}}(N_{\mathrm{ZF}} - \frac{\alpha}{4})^{1-\frac{\alpha}{2}}\mathbb{E}\{W\}}{\alpha - 2}$$

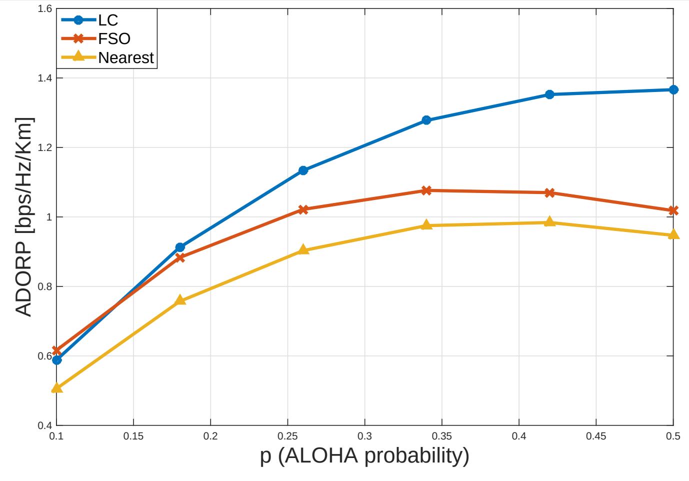
ADORP vs p_{tx}

Multiple antenna, $N_{\mathrm{ZF}} = 1, N_R = 10, \alpha = 3$



ADORP vs p_{tx}

Multiple antenna, $N_{\mathrm{ZF}} = 2, N_R = 10, \alpha = 3$

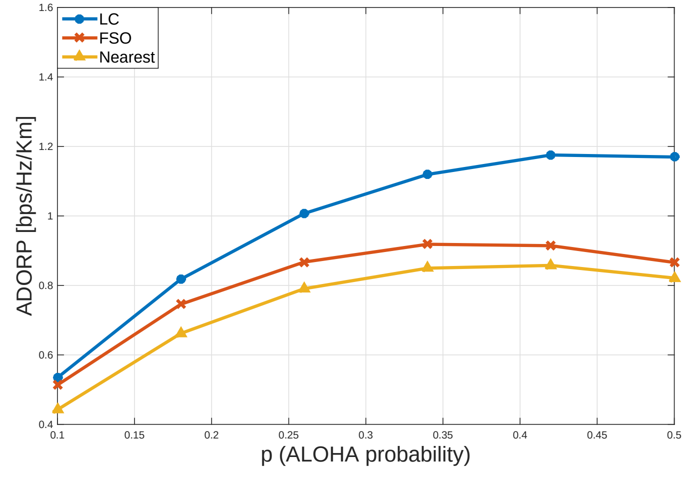


ADORP vs p_{tx}

Optimal and Suboptimal Routing

- Introduction
- System Model
- Single Antenna
- **Multiple Antennas**
- Conclusions

Multiple antenna, $N_{\mathrm{ZF}} = 3$, $N_R = 10$, $\alpha = 3$



Conclusions

Conclusions

- Presented novel routing metrics.
- Based on optimization of the ADORP bound.
- Simple to evaluate locally.
- Uses only local knowledge and exploits the statistics of the interference.
- Close to optimal, outperforms traditional schemes.

Thank you!

Questions?

ERD Lower Bound

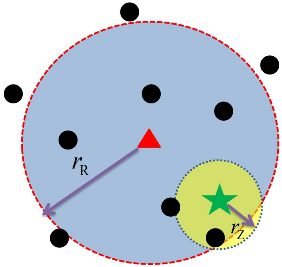
- Rewrite

$$\bar{D}(f(\cdot)) = \lambda p \mathbb{E}_{\mathcal{M}}\left\{ G\left(f(\mathcal{M}), \mathcal{M}\right)\right\}$$

where

$$G(i, \mathcal{M}) = \mathbb{E}_{J|\mathcal{M}}\left\{ r_{i,0} \log_2\left(1 + \frac{\rho S_i}{J_i}\right) \Big| \mathcal{M}\right\}.$$

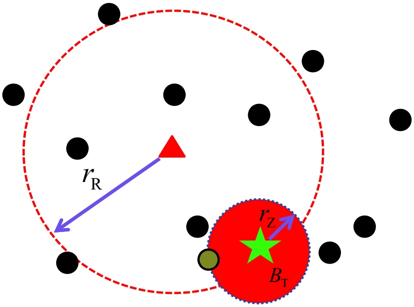
- **Routing Zone** and **Threshold Zone**



ERD Lower Bound

- $p_{\mathrm{Z}}(i, \mathcal{M})$: probability that no transmitter within distance r_{Z} from node i:

$$p_{\mathrm{Z}}(i, \mathcal{M}) = \begin{cases} (1-p)^{N_{\mathrm{Z},i}}, & \text{if } \|\mathbf{r}_i\| + r_{\mathrm{Z}} < r_{\mathrm{R}} \\ e^{-\lambda p B_{\mathrm{T},i}} (1-p)^{N_{\mathrm{Z},i}}, & \text{o.w.} \end{cases}$$



ERD Lower Bound

- Lower bound

$$G(i,\mathcal{M}) = (1-p_{\mathrm{Z}}(i,\mathcal{M}))\mathbb{E}\left\{r_i \log_2\left(1+\frac{\rho\cdot S_i}{J_i}\right)\Big| r_{\min,i} \le r_{\mathrm{Z}}, \mathcal{M}\right\}$$
$$+ p_{\mathrm{Z}}(i,\mathcal{M})\mathbb{E}\left\{r_i \log_2\left(1+\frac{\rho\cdot S_i}{J_i}\right)\Big| r_{\min,i} > r_{\mathrm{Z}}, \mathcal{M}\right\}$$
$$\ge p_{\mathrm{Z}}(i,\mathcal{M})\mathbb{E}\left\{r_i \log_2\left(1+\frac{\rho\cdot S_i}{J_i}\right)\Big| r_{\min,i} > r_{\mathrm{Z}}, \mathcal{M}\right\}$$
$$= p_{\mathrm{Z}}(i,\mathcal{M})\cdot r_i \log_2\left(1+\frac{\rho\cdot S_i}{\mathbb{E}\{J_i | r_{\min,i} > r_{\mathrm{Z}}, \mathcal{M}\}}\right)$$

- Eventually,

$$\mathbb{E}\{J_i | r_{\min,i} > r_{\mathrm{Z}}, \mathcal{M}\} = \bar{J}_1^i + \bar{J}_2^i + \bar{J}_3^i$$

Partial Zero Forcing (PZF)

- Cancels its N_{ZF} nearest transmitters.
- T_{ZF}: # of inter-streams to be canceled.
- Remaining degrees of freedom (DOF)

$$L = N_{\mathrm{R}} - T_{\mathrm{ZF}} - (K_0 - 1).$$

- Set of N_{ZF} indices of undesired transmitters

$$\mathcal{N}_{\mathrm{ZF}}^{\mathrm{Inter}} = \{j \in \Phi_T : r_j \leq r_{\mathrm{ZF}}\}.$$

- Set of its $(K-1)$ intra-streams

$$\mathcal{N}_{\mathrm{ZF},k}^{\mathrm{Intra}} = \{1, 2, ..., k-1, k+1, ..., K\}.$$

Continue