TELECOM ParisTech

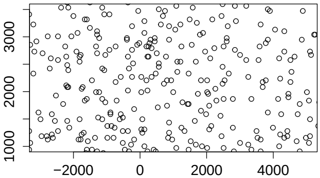
Institut Mines-Telecom

Stein-Dirichlet-Malliavin method and applications

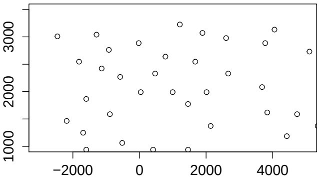
L. Decreusefond

Simons conference

BTS deployement



Paris, all frequency bands



1 frequency band

Statistical analysis

Tests [Gomez et al.]

Locations of all BTS $\simeq$ Poisson point process

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Questions

- Which model for one frequency band ?

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- Which model for one frequency band ?
- Explain/quantify why the superposition is Poisson

Related works

- Deng, Zhou, Haenggi. The Ginibre Point Process as a Model for Wireless Networks with Repulsion
- Li, Baccelli, Dhillon, Andrews. Statistical modeling and probabilistic analysis of cellular networks with DPP

Ginibre process

Random matrices

$$\omega = \lim_{\text{size} \to \infty} \text{Eigenvalues of} \begin{pmatrix} \mathcal{N}_{\mathbf{C}}(0,1) & \dots & \dots & \mathcal{N}_{\mathbf{C}}(0,1) \\ \vdots & & & \vdots \\ \mathcal{N}_{\mathbf{C}}(0,1) & \dots & \dots & \mathcal{N}_{\mathbf{C}}(0,1) \end{pmatrix}$$

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Definition (Correlation functions)

$$\rho_n(x_1, \cdots, x_n) = \det\Big(K(x_i, x_j),\ 1 \le i,j \le n\Big)$$

where

$$K(x,y) = \frac{1}{\pi}\exp\Big(x\bar{y} - \frac{1}{2}(|x|^2 + |y|^2)\Big),\ \ x, y \in \mathbf{C}$$

Randomization

(λ, β)-Ginibre

- Apply a β-thinning

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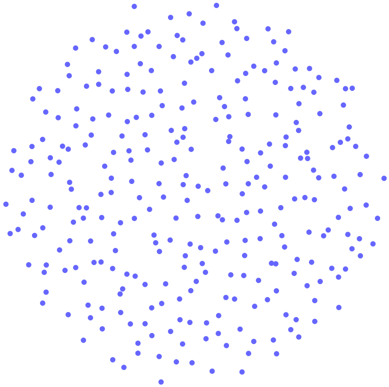
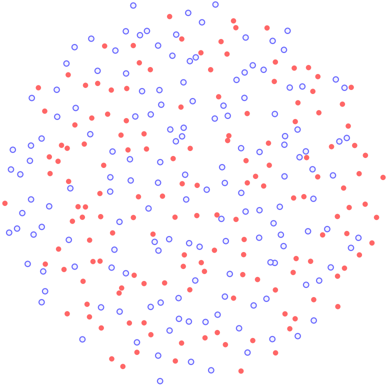
(λ, β)-Ginibre

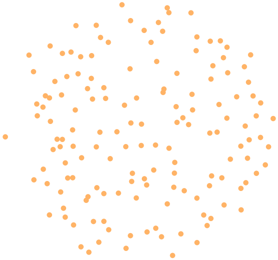
- Apply a β-thinning
- Apply a dilation of $\lambda\sqrt{\beta}$

Poisson as a Ginibre

$$(\lambda, \beta) - \text{Ginibre} \xrightarrow{\beta \to 0} \text{Poisson}(\pi^{-1}\lambda \mathrm{d}y)$$







Results [Gomez et al.]

Table: Numerical values of β and λ per technology and operator

		Orange	SFR	Bouygues	Free
GSM 900	β	0.81	0.76	0.65	NA
	λ	2.39	2.65	2.63	NA
GSM 1800	β	0.84	0.85	0.71	NA
	λ	3.00	2.39	3.59	NA
LTE 800	β	1.00	0.93	0.67	NA
	λ	0.67	1.65	1.87	NA
LTE 2600	β	0.93	0.67	0.63	0.89
	λ	2.80	2.76	2.46	1.05

Topology of configuration space

Definition

A configuration is a locally finite set of particles on a Polish space $\mathbb{Y}$

$$\int f \, \mathrm{d}\omega = \sum_{x \in \omega} f(x)$$

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Vague topology

$$\omega_n \xrightarrow{\text{vaguely}} \omega \Longleftrightarrow \int f \mathrm{d}\omega_n \xrightarrow{n \to \infty} \int f \mathrm{d}\omega$$

for all f continuous with compact support from $\mathbb{Y}$ to $\mathbb{R}$

Lipschitz functionals

Distance between configurations

$c(\omega, \eta) = \mathsf{dist}_{\mathsf{TV}}(\omega, \eta) =$ number of different points

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Definition

$F : \mathfrak{N}_{\mathbb{Y}} \to \mathbf{R}$ is TV–Lip_1 if

$$|F(\omega) - F(\eta)| \le \mathrm{dist}_{\mathrm{TV}}(\omega, \eta)$$

Example : $\omega \longmapsto \omega(A)$

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$$\text{Example : } \omega \longmapsto \omega(A)$$

Definition (Kantorovitch-Rubinstein distance)

$$\text{d}_{\text{KR}}(\mathbf{P}, \mathbf{Q}) := \sup_{F \in \text{TV–Lip}_1} \left(\mathbf{E}_{\mathbf{P}}[F] - \mathbf{E}_{\mathbf{Q}}[F] \right),$$

Convergence in configuration space

Theorem (LD-Schulte-Thäle)

$$\mathrm{d}_{KR}(\mathbf{P}_n, \mathbf{Q}) \xrightarrow{n\to\infty} 0 \Longrightarrow \mathbf{P}_n \xrightarrow{distr.} \mathbf{Q}$$

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where

$$\mathbf{P}_n \xrightarrow{distr.} \mathbf{Q} \Longleftrightarrow \int F \; d\mathbf{P}_n \xrightarrow{F \text{ vaguely cont.}} \int F \; d\mathbf{Q}$$

Framework

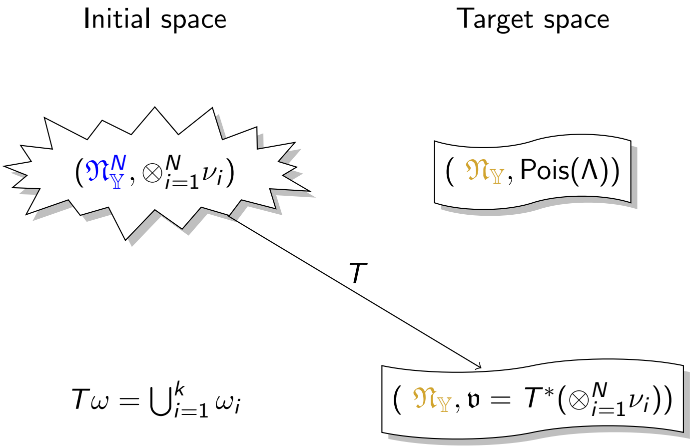
Initial space

$(\mathfrak{N}_{\mathbb{Y}}^{N}, \otimes_{i=1}^{N} \nu_i)$

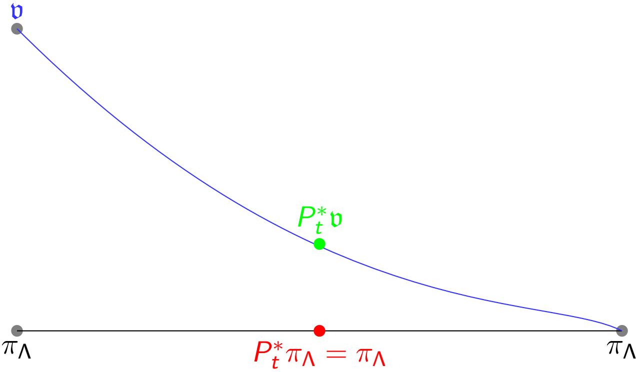
Target space

$(\mathfrak{N}_{\mathbb{Y}}, \mathrm{Pois}(\Lambda))$

Framework


Initial space
Target space
$(\mathfrak{N}_{\mathbb{Y}}^{N}, \otimes_{i=1}^{N} \nu_i)$
$(\mathfrak{N}_{\mathbb{Y}}, \text{Pois}(\Lambda))$
T
$T\omega = \bigcup_{i=1}^{k} \omega_i$
$(\mathfrak{N}_{\mathbb{Y}}, \mathfrak{v} = T^*(\otimes_{i=1}^{N} \nu_i))$

Homotopy



$\mathfrak{v}$
$P_t^*\mathfrak{v}$
π_Λ
$P_t^*\pi_\Lambda = \pi_\Lambda$
π_Λ

Stein's method

The main tool

Construct a Markov process $(X(s), s \geq 0)$

- with values in $\mathfrak{N}_{\mathbb{Y}}$
- ergodic with $\text{Pois}(\Lambda)$ as invariant distribution

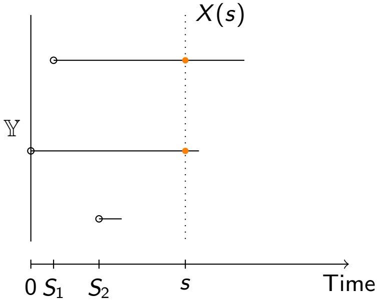
$$X(s) \xrightarrow{distr.} \text{Pois}(\Lambda) := \pi_\Lambda$$

  for all initial condition $X(0)$
- for which π_Λ is a stationary distribution

$$X(0) \overset{\text{distr.}}{=} \pi_\Lambda \Longrightarrow X(s) \overset{\text{distr.}}{=} \pi_\Lambda, \ \forall s > 0$$

- Equivalently $\int LF \text{d}m = 0, \ \forall F$ iff $m = \pi_\Lambda$
- Equivalently $P_t^* \pi_\Lambda = \pi_\Lambda$ where $P_t = e^{tL}$

Realization of a Glauber process



- $S_1, S_2, \cdots$: Poisson process of intensity $\Lambda(\mathbb{Y})\, ds$
- Lifetimes : Exponential rv of param. 1
- Remark : Nb of particles $\sim$ M/M/∞

Poisson point process

PPP over $\mathbb{Y}$

- Generator

$$LF(\omega) := \int_{\mathbb{Y}} \Big(F(\omega \cup \{y\}) - F(\omega) \Big) \mathrm{d}\Lambda(y) + \sum_{y \in \omega} F(\omega \setminus \{y\}) - F(\omega)$$

- Dist. $X(t)$=PPP$((1 - e^{-t})\Lambda) + e^{-t}$-thinning of the I.C.

$$P_t F(\omega) = \mathbf{E}\left[F(X(t)) \,|\, X(0) = \omega \right]$$

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- Generator

$$LF(\omega) := \int_{\mathbb{Y}} \Big(F(\omega \cup \{y\}) - F(\omega) \Big) \Lambda(\mathbb{Y}) \frac{\mathrm{d}\Lambda(y)}{\Lambda(\mathbb{Y})}$$
$$+ \omega(\mathbb{Y}) \sum_{y \in \omega} F(\omega \setminus \{y\}) - F(\omega) \frac{1}{\omega(\mathbb{Y})}$$

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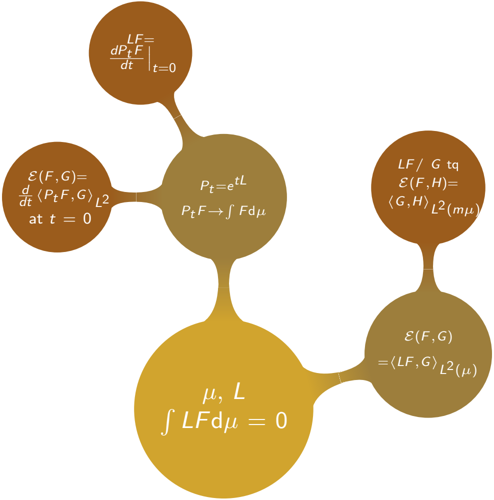
Dirichlet structure



Stein representation formula

Kantorovitch-Rubinstein distance between $\mathfrak{v}$ and μ

$$P_\infty F(\omega) - P_0 F(\omega) = \int_0^\infty \frac{d}{dt} P_t F(\omega) \mathrm{d}t$$

Stein representation formula

Kantorovitch-Rubinstein distance between $\mathfrak{v}$ and μ

$$P_\infty F(\omega) - F(\omega) = \int_0^\infty L P_t F(\omega) \mathrm{d}t$$

Stein representation formula

Kantorovitch-Rubinstein distance between $\mathfrak{v}$ and μ

$$\int_{\mathfrak{N}_{\mathbb{Y}}} F \mathrm{d}\mu - F(\omega) = \int_0^\infty L P_t F(\omega) \mathrm{d}t$$

Stein representation formula

Kantorovitch-Rubinstein distance between $\mathfrak{v}$ and μ

$$\int_{\mathfrak{N}_{\mathbb{Y}}} F \mathrm{d}\mu - \int_{\mathfrak{N}_{\mathbb{Y}}} F(\omega)\ \mathrm{d}\mathfrak{v}(\omega) = \int_{\mathfrak{N}_{\mathbb{Y}}} \int_0^{\infty} LP_t F(\omega) \mathrm{d}t\ \mathrm{d}\mathfrak{v}(\omega)$$

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$$LP_tF(\omega) := \int_{\mathbb{Y}} \Big(P_tF(\omega \cup \{y\}) - P_tF(\omega) \Big)\ \mathrm{d}\Lambda(y) + \sum_{y \in \omega} P_tF(\omega \backslash \{y\}) - P_tF(\omega)$$

Gen. IPP

Integrate by parts, you should

Definition (Difference operator)

For $F : \mathfrak{N}_{\mathbb{Y}} \to \mathbf{R}$,

$$\begin{array}{rccc} DF: & \mathbb{Y} \times \mathfrak{N}_{\mathbb{Y}} & \longrightarrow & \mathbf{R} \\ & (y, \omega) & \longmapsto & D_y F(\omega) = F(\omega \cup \{y\}) - F(\omega \backslash \{y\}). \end{array}$$

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Campbell-Mecke formula $\Longleftrightarrow$ IPP

$$\mathbf{E}\left[\sum_{y \in \omega} f(y, \omega)\right] = \mathbf{E}\left[\int_{\mathbb{Y}} f(y, \omega \cup \{y\}) \mathrm{d}\Lambda(y)\right]$$

is equivalent to

$$\mathbf{E}\left[\int_{\mathbb{Y}} D_y U(\omega)\ f(y, \omega) \mathrm{d}\Lambda(y)\right] \\ = \mathbf{E}\left[U(\omega) \int_{\mathbb{Y}} f(y, \omega)(\mathrm{d}\omega(y) - \mathrm{d}\Lambda(y))\right]$$

Divergence

Consequence of Campbell-Mecke formula

$$\mathbf{E}\left[\int_{\mathbb{Y}} D_y U(\omega) f(y,\omega)\,\Lambda(\mathrm{d}y)\right] = \mathbf{E}\left[U(\omega)\int_{\mathbb{Y}} f(y,\omega)(\mathrm{d}\omega(y) - \Lambda(\mathrm{d}y))\right]$$

means that

$$D^* f(\omega) = \int_{\mathbb{Y}} f(y,\omega)(\mathrm{d}\omega(y) - \Lambda(\mathrm{d}y))$$

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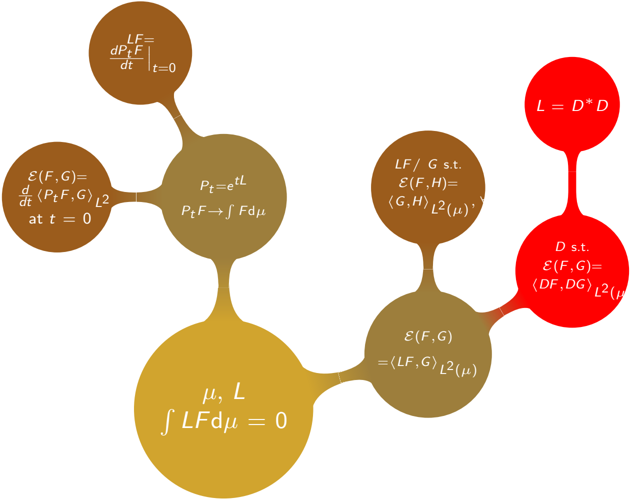
Generator

Dirichlet-Malliavin structure



For general point processes

Definition (Reduced Campbell measure)

$$C_{\mathbf{P}}(A \times B) = \int \sum_{x \in \omega} \mathbf{1}_A(x) \mathbf{1}_B(\omega \backslash x) \, \mathrm{d}\mathbf{P}(\omega),$$

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Definition (Papangelou intensity)

Assume that $C_{\mathbf{P}} \ll \Lambda \otimes \mathbf{P}$. Then,

$$c(x, \omega) = \frac{dC_{\mathbf{P}}}{d\Lambda \otimes d\mathbf{P}}(x, \omega)$$

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Informally

$$c(x, \omega) \sim \mathbf{P}(\omega \cup \{x\} \,|\, \omega)$$

Examples

Poisson process : $\mathbf{P} = \pi_\Lambda$

$$c(x, \omega) = 1$$

Examples

Poisson process : $\mathbf{P} = \pi_\Lambda$

$$c(x,\, \omega) = 1$$

Pairwise interactions Gibbs process

If

$$\mathrm{d}\mathbf{P}(\omega) = \exp(-\sum_{x,y\in\omega} \phi(x,\, y))\ \mathrm{d}\pi_\Lambda(\omega)$$

Then

$$c(x,\, \omega) = \exp(-\sum_{y\in\omega} \phi(x,\, y))$$

Ginibre process

Lemma

Functions $(z^n \exp(-|z|^2/2), n \geq 0)$ *are orthogonal on* $L^2(B(0; R), dz = dx.dy)$

Ginibre process

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Definition

$$\alpha_n^2 = \int_{B(0;R)} |z|^{2n} \exp(-|z|^2) \mathrm{d}z = \frac{\pi}{2} \int_0^{R^2} u^n\, e^{-u} \mathrm{d}u$$

$$J(z, w) = \sum_{n=0}^{\infty} \frac{\alpha_n}{1-\alpha_n} \frac{z^n \exp(-|z|^2/2)}{\alpha_n} \frac{\overline{w}^n \exp(-|w|^2/2)}{\alpha_n}$$

$$c(z_0, \{z_1, \cdots, z_n\}) = \frac{\det(J(z_a, z_b), 0 \leq a, b \leq n)}{\det(J(z_a, z_b), 1 \leq a, b \leq n)}$$

Integration by parts= GNZ formula

Theorem (Georgii-Nguyen-Zessin (GNZ) formula)

$$\int_{\mathfrak{N}_{\mathbb{Y}}} \sum_{y\in\omega} u(y,\omega\backslash\{y\})\mathrm{d}\mathbf{P}(\omega) = \int_{\mathfrak{N}_{\mathbb{Y}}} \int_{\mathbb{Y}} u(y,\omega)\, c(y,\omega)\mathrm{d}\Lambda(y)\mathrm{d}\mathbf{P}(\omega)$$

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Theorem (LD-Flint)

For any bounded function F on $\mathfrak{N}_{\mathbb{Y}}$,

$$\mathbf{E}\left[F(\omega)\,\left(\int_{\mathbb{Y}} u(y,\omega\backslash y)\mathrm{d}\omega(y) - \int_{\mathbb{Y}} u(y,\omega)\, c(y,\omega)\mathrm{d}\Lambda(y)\right)\right]$$
$$= \mathbf{E}[\int_{\mathbb{Y}} D_y F(\omega)\ u(y,\omega)c(y,\omega)\mathrm{d}\Lambda(y)]$$

General point process

Stein-Dirichlet-Malliavin structure

- **P**=dist. of point process of Papangelou intensity c
- Generator

$$LF(\omega) := \int_{\mathbb{Y}} D_y F(\omega) c(y,\omega) \, \mathrm{d}\Lambda(y) + \sum_{y\in\omega} F(\omega\backslash\{y\}) - F(\omega)$$

- X the associated Glauber process
- $L = D^* D$

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$$\int_{\mathfrak{N}_{\mathbb{Y}}} F \mathrm{d}\mu - \int_{\mathfrak{N}_{\mathbb{Y}}} F(\omega)\, \mathrm{d}\mathfrak{v}(\omega) = \int_{\mathfrak{N}_{\mathbb{Y}}} \int_0^\infty LP_t F(\omega) \mathrm{d}t\, \mathrm{d}\mathfrak{v}(\omega)$$

$$LP_t F(\omega) := \int_{\mathbb{Y}} D_y P_t F(\omega)\, \mathrm{d}\Lambda(y) + \sum_{y \in \omega} P_t F(\omega \setminus \{y\}) - P_t F(\omega)$$

Our situation

Initial space

- ν_i= PP of reference measure Λ_i and PI $c_i^{\Lambda_i}$

Target space

- $\mathbf{P}$= Poisson(Λ) with $\Lambda = \sum_{i=1}^{N} \Lambda_i$

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- $T(\omega_1, \cdots, \omega_N) = \bigcup_{i=1}^N \omega_i$

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Target space

- $\mathbf{P} =$ Poisson(Λ) with $\Lambda = \sum_{i=1}^N \Lambda_i$
- $\mathfrak{v} = T^* \otimes_{i=1}^N \nu_i$ has PI

$$c^\Lambda(y, \cup_{i=1}^N \omega_i) = \sum_{i=1}^N c_i^\Lambda(y, \omega_i)$$
$$= \sum_{i=1}^N c_i^{\Lambda_i}(y, \omega_i) \frac{\mathrm{d}\Lambda_i}{\mathrm{d}\Lambda}(y)$$

Second step

$$\int_{\mathfrak{N}_{\mathbb{Y}}} \sum_{y\in\omega} P_tF(\omega\backslash\{y\}) - P_tF(\omega)\mathrm{d}\mathfrak{v}(\omega)$$

$$= -\int_{\mathfrak{N}_{\mathbb{Y}}} \sum_{y\in\omega} D_yP_tF(\omega)\ c(y,\omega)\mathrm{d}\Lambda(y)\mathrm{d}\mathfrak{v}(\omega)$$

$$= -\sum_{i=1}^{N} \int_{\mathfrak{N}_{\mathbb{Y}}} \int_{\mathbb{Y}} D_yP_tF(\cup_{i=1}^{N}\omega_i)\ c_i^{\Lambda}(y,\omega_i)\ \mathrm{d}\Lambda(y) \otimes_{i=1}^{N} \mathrm{d}\nu(\omega_i)$$

Putting all together

Partial conclusion

$$\int_{\mathfrak{N}_{\mathbb{Y}}} \int_0^{\infty} LP_tF(\omega)\,\mathrm{d}t\,\mathrm{d}\mathfrak{v}(\omega)$$
$$= \mathbf{E}\left[\int_0^{\infty}\int_{\mathbb{Y}} D_yP_tF(\cup_{i=1}^N\omega_i)\left|1-\sum_{i=1}^N c_i^{\Lambda}(x,\omega_i)\right|\,\mathrm{d}\Lambda(y)\,\mathrm{d}t\right]$$

Putting all together

Partial conclusion

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$$= \mathbf{E}\left[\int_0^\infty \int_{\mathbb{Y}} e^{-t} P_t D_y F(\cup_{i=1}^N \omega_i) \left| 1 - \sum_{i=1}^N c_i^\Lambda(x, \omega_i) \right| \mathrm{d}\Lambda(y)\, \mathrm{d}t \right]$$

Putting all together

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Theorem

$$d_R(\mathfrak{v}, \pi_\Lambda) \le \mathbf{E}\left[\int_{\mathbb{Y}} \left| 1 - \sum_{i=1}^N c_i^{\Lambda}(y, \omega_i) \right| d\Lambda(y) \right]$$

Superposition

Theorem (LD-A. Vasseur)

$$\mathrm{d}_{KR}\left(\bigoplus_{i=1}^{N}(\lambda_{N,i},\beta_{N,i})\text{-}\mathit{Ginibre},\ \ \mathit{Poisson}(\pi^{-1}\sum_{i=1}^{N}\lambda_{N,i}\mathrm{d}y)\right) \le \frac{c}{N}\ \sup_i \beta_{N,i}$$

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$$\mathrm{d}_{KR}\Big((\lambda,\beta)-\mathit{Ginibre}, \mathit{Poisson}(\pi^{-1}\lambda)\Big) \leq c\beta$$

Repulsive point processes

Definition (Repulsivity)

$$\eta \subset \omega \Longrightarrow c(x,\omega) \le c(x,\eta)$$

Repulsive point processes

Definition (Repulsivity)

$$\eta \subset \omega \Longrightarrow c(x,\omega) \le c(x,\eta)$$

Weak repulsivity

$$c(x,\omega) \le c(x,\emptyset)$$

Convergence of weak repulsive processes

Theorem

If $\nu_{N,i},\ i = 1, \cdots, N$ *are w-repulsive*

$$\mathrm{d}_{KR}\left(T^*(\otimes_{i=1}^N \nu_{N,i}), \mathit{Poisson}(\Lambda)\right) \le \int_{\mathbb{Y}} \left| \sum_{i=1}^N \mathbf{E}\left[c^\Lambda_{N,i}(y,\omega)\right] - 1 \right| \mathrm{d}\Lambda(y) + 2\sum_{i=1}^N (1-\nu_{N,i}(\{\emptyset\}))^2$$

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure

Initial space

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure

Initial space

- Gradient and its adjoint for the initial measure

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure
- $|DP_tF| \le \psi(t)P_t|DF|$ with $\psi \in L^1(\mathbf{R}^+)$

Initial space

- Gradient and its adjoint for the initial measure

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure
  Characterization of the target measure
- $|DP_tF| \leq \psi(t)P_t|DF|$ with $\psi \in L^1(\mathbf{R}^+)$

Initial space

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Conclusion

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Initial space

- Gradient and its adjoint for the initial measure
  Exchangeable pairs, bias coupling, etc

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure
  Characterization of the target measure
- $|DP_tF| \leq \psi(t)P_t|DF|$ with $\psi \in L^1(\mathbf{R}^+)$
  Properties of the solution of the Stein equation, gradient bounds

Initial space

- Gradient and its adjoint for the initial measure
  Exchangeable pairs, bias coupling, etc

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