How performance metrics depend on the traffic demand in large cellular networks

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Introduction

- Performance metrics in cellular data networks
  - cell loads, users number per cell, average user's throughput
- They depend on
  - traffic demand⇒ Dynamics of call arrivals and departures
  - base stations (BS) positioning
    - irregularity⇒ Performance varies across cells
  - inter-cell interference⇒ Performance in different cells are interdependent
- In this work we propose
  - an analytic approach accounting for the above three aspects in the evaluation of the performance metrics in large irregular cellular networks
  - validated by measurements performed in operational networks

Network geometry and propagation

- Base stations (BS) locations modelled by a point process $\Phi = \{X_n\}_{n\in\mathbb{Z}}$ on $\mathbb{R}^2$
  - assumed stationary, simple and ergodic
  - with intensity parameter $\lambda > 0$
- BS X_n emits a power $P_n > 0$ such that $\{P_n\}_{n\in\mathbb{Z}}$ are marks of Φ
- Propagation loss comprises
  - a deterministic effect depending on the relative location $y - X_n$ of the receiver with respect to transmitter ; that is a measurable mapping

$$l : \mathbb{R}^2 \to \mathbb{R}_+$$

  - and a random effect called shadowing
    - The shadowing between BS X_n and all the locations $y \in \mathbb{R}^2$ is modelled by a measurable stochastic process $S_n(y - X_n)$ with values in $\mathbb{R}_+$
    - the processes $\{S_n(\cdot)\}_{n\in\mathbb{Z}}$ are marks of Φ

Network geometry and propagation

- The power received at location y from BS X_n is

$$\frac{P_n S_n \left(y - X_n\right)}{l \left(y - X_n\right)}, \quad y \in \mathbb{R}^2, n \in \mathbb{Z}$$

  - Its inverse is denoted by $L_{X_n}\left(y\right)$
- The signal-to-interference-and-noise (SINR) power ratio in the downlink for a user located at y served by BS X equals

$$\mathrm{SINR}\left(y, \Phi\right) = \frac{1/L_X\left(y\right)}{N + \sum_{Y \in \Phi \setminus \{X\}} \varphi_Y / L_Y\left(y\right)}$$

  - where $N \geq 0$ is the noise power
  - $\{\varphi_Y\}_{Y \in \Phi}$ are additional (not necessarily independent) marks in $\mathbb{R}_+$ of the point process Φ called interference factors

Service model

- Each BS $X \in \Phi$ serves the locations where the received power is the strongest among all the BS ; that is

$$V(X) = \left\{y \in \mathbb{R}^2 : L_X(y) \leq L_Y(y) \text{ for all } Y \in \Phi\right\}$$

  called cell of X
- A single user served by BS X and located at $y \in V(X)$ gets a bit-rate

$$R(\mathrm{SINR}(y, \Phi))$$

  called peak bit-rate
  - Particular form of this (measurable) function $R : \bar{\mathbb{R}}_+ \to \bar{\mathbb{R}}_+$ depends on the actual technology used to support the wireless link
- Each user in a cell gets an equal portion of time for his service.
  - Thus when there are k users in a cell $y_1, y_2, \ldots, y_k \in V(X)$, each one gets a bit-rate equal to his peak bit-rate divided by k
  - i.e. the bit-rate of user located at y_j equals $\frac{1}{k} R(\mathrm{SINR}(y_j, \Phi))$, $j \in \{1, 2, \ldots, k\}$

Traffic model

- There are γ arrivals per surface unit and per time unit
- Variable bit-rate (VBR) traffic : at their arrival, users require to transmit some volume of data at a bit-rate decided by the network
  - Each user arrives at a location uniformly distributed and requires to download a random volume of data of mean $1/\mu$ bits
- Arrival locations, inter-arrival durations as well as the data volumes are assumed independent
- Users don't move during their calls
- Traffic demand per surface unit

$$\rho = \frac{\gamma}{\mu} \text{ bit/s/km}^2$$

- The traffic demand in cell $X \in \Phi$ equals

$$\rho(X) = \rho |V(X)| \text{ bit/s}$$

Cell performance metrics [1]

- Service in cell $V(X)$ is stable when

$$\rho(X) < \rho_{\mathrm{c}}(X) := \frac{|V(X)|}{\int_{V(X)} 1/R(\mathrm{SINR}(y,\Phi))\,dy}$$

called critical traffic : harmonic mean of the peak bit-rate. In case of stability,

  - User's throughput

$$r(X) = \max(\rho_{\mathrm{c}}(X) - \rho(X), 0)$$

  - Number of users

$$N(X) = \frac{\rho(X)}{r(X)}$$

  - Probability that BS is not idling equals $\min(\theta(X), 1)$ where

$$\theta(X) := \frac{\rho(X)}{\rho_{\mathrm{c}}(X)} = \rho\int_{V(X)} 1/R(\mathrm{SINR}(y,\Phi))\,dy$$

called cell load

Typical cell

- Are there global metrics of the network allowing to characterize its macroscopic behaviour ?
- Consider spatial averages of the cell characteristics over an increasing network window A
- By the ergodic theorem of point processes (discrete version), these averages converge to Palm-expectations of the respective characteristics of the "typical cell" $V(0)$
  - For example, for traffic demand

$$\lim_{|A|\to\infty} \frac{1}{\Phi(A)} \sum_{X\in A} \rho(X) = \mathbf{E}^0[\rho(0)]$$

    and for cell load

$$\lim_{|A|\to\infty} \frac{1}{\Phi(A)} \sum_{X\in A} \theta(X) = \mathbf{E}^0[\theta(0)]$$

  - Analoguous convergence holds for other cell characteristics : critical traffic, user's throughput, number of users

Typical cell characteristics

- Technical condition : Assume that location 0 belongs to a unique cell a.s.
- Then by the inverse formula of Palm calculus, typical cell traffic demand

$$\mathbf{E}^0[\rho(0)] = \frac{\rho}{\lambda}$$

  and cell load

$$\mathbf{E}^0[\theta(0)] = \frac{\rho}{\lambda}\mathbf{E}\left[\frac{1}{R\left(\mathrm{SINR}\left(0,\Phi\right)\right)}\right]$$

- Right-hand side : Expectation of the inverse of the peak bit-rate of the *typical user* with respect to the *stationary* distribution of Φ
  - By the ergodic theorem of point processes (continuous version)

$$\mathbf{E}\left[\frac{1}{R\left(\mathrm{SINR}\left(0,\Phi\right)\right)}\right] = \lim_{|A|\to\infty}\frac{1}{|A|}\int_A \frac{1}{R\left(\mathrm{SINR}\left(y,\Phi\right)\right)}\mathrm{d}y$$

Cell-load equations

- The above results hold true
  - whatever is the point process Φ of BS locations provided it is simple stationary and ergodic (not necessarily Poisson)
  - whatever are the marks $\{\varphi_Y\}_{Y\in\Phi}$ pondering the interference
- In real networks a BS transmits only when it serves at least one user, thus we take φ_Y equal to the probability that Y is not idling

$$\varphi_Y = \min\left(\theta\left(Y\right), 1\right)$$

Then

$$\mathrm{SINR}\left(y, \Phi\right) = \frac{1/L_X\left(y\right)}{N + \sum_{Y\in\Phi\setminus\{X\}} \min\left(\theta\left(Y\right), 1\right)/L_Y\left(y\right)}$$

- Recalling the expression of the cell load

$$\theta\left(X\right) = \rho \int_{V(X)} 1/R\left(\mathrm{SINR}\left(y, \Phi\right)\right) dy$$

we see that cell loads $\theta(X)$ are related to each other by a system of cell-load equations

Average user's throughput

- Define the average user's throughput in the network as the ratio of mean volume of data request to mean service duration

$$r^0 := \lim_{|A|\to\infty} \frac{1/\mu}{\text{mean service time in } A \cap \mathcal{S}}$$

  where $\mathcal{S}$ is the union of stable cells
- By Little's law and ergodic theorem, it is shown in [2] that

$$r^0 = \frac{\rho}{\lambda} \frac{\mathbf{P}(0 \in \mathcal{S})}{N^0}$$

  where $N^0 := \mathbf{E}^0[N(0)\mathbf{1}\{N(0) < \infty\}]$
- N^0 and $\mathbf{P}(0 \in \mathcal{S})$ do not have explicit analytic expressions !

Mean cell model

- Virtual cell defined as a queue having the same traffic demand and load as the typical cell ; that is

$$\bar{\rho} := \mathbf{E}^0[\rho(0)] = \frac{\rho}{\lambda}$$

$$\bar{\theta} := \mathbf{E}^0[\theta(0)] = \frac{\rho}{\lambda}\mathbf{E}\left[\frac{1}{R\left(\mathrm{SINR}\left(0,\Phi\right)\right)}\right]$$

- Remaining characteristics are related to the above two via the relations of cell performance metrics
  - critical traffic demand

$$\rho_{\mathrm{c}}\left(X\right) = \frac{\rho\left(X\right)}{\theta\left(X\right)} \rightarrow \bar{\rho}_{\mathrm{c}} := \frac{\bar{\rho}}{\bar{\theta}}$$

  - user's throughput

$$r\left(X\right) = \max(\rho_{\mathrm{c}}\left(X\right) - \rho\left(X\right), 0) \rightarrow \bar{r} := \max\left(\bar{\rho}_{\mathrm{c}} - \bar{\rho}, 0\right)$$

  - number of users

$$N\left(X\right) = \frac{\rho\left(X\right)}{r\left(X\right)} \rightarrow \bar{N} := \frac{\bar{\rho}}{\bar{r}}$$

Mean cell load equation

- Assume that all BS emit at the same power
- In the mean cell model, we consider the following (single) equation in the mean-cell load $\bar{\theta}$

$$\bar{\theta} = \frac{\rho}{\lambda}\mathbf{E}\left[1/R\left(\frac{1/L_{X^*}(0)}{N + \min\left(\bar{\theta}, 1\right)\sum_{Y\in\Phi\setminus\{X^*\}} 1/L_Y(0)}\right)\right]$$

where X^* is the location of the BS whose cell covers the origin.

  - We solve the above equation with $\bar{\theta}$ as unknown
  - We will see in the numerical section that the solution of this equation gives a good estimate of the empirical average of the loads $\{\theta(X)\}_{X\in\Phi}$ obtained by solving the system of cell-load equations for the typical cell model

Scaling laws for homogeneous networks

- Consider a homogeneous network model with a deterministic propagation loss of the form

$$l(x) = (K|x|)^{\beta}, \quad x \in \mathbb{R}^2$$

where $K > 0$ and $\beta > 2$ are two given parameters

- For $\alpha > 0$ consider a network obtained from this original one by scaling
  - the base station locations $\Phi' = \{X' = \alpha X\}_{X \in \Phi}$,
  - the traffic demand intensity $\rho' = \rho/\alpha^2$,
  - distance coefficient $K' = K/\alpha$
  - and shadowing processes $S'_n(y) = S_n\left(\frac{y}{\alpha}\right)$,

  while preserving the original powers $P'_n = P_n$
- For the rescaled network consider the cells $V'(X')$ and their characteristics $\rho'(X')$, $\rho'_{\mathrm{c}}(X')$, $r'(X')$, $N'(X')$, $\theta'(X')$

Scaling laws for homogeneous networks

- *Proposition* : Assume $\varphi'_{X'} = \varphi_X$, $X \in \Phi$. Then for any $X' \in \Phi'$, we have $V'(\alpha X_n) = \alpha V(X_n)$ while $\rho'(X') = \rho(X)$, $\rho'_{\mathrm{c}}(X') = \rho_{\mathrm{c}}(X)$, $r'(X') = r(X)$, $N'(X') = N(X)$, $\theta'(X') = \theta(X)$
- *Corollary* : Assume $\varphi_X = \min(\theta(X), 1)$, $\varphi'_X = \min(\theta'(X), 1)$. Then the load equations are the same for the two networks Φ and Φ'. Therefore $\theta'(X') = \theta(X)$, $X \in \Phi$ and by above Proposition, $\rho'_{\mathrm{c}}(X') = \rho_{\mathrm{c}}(X)$, $r'(X') = r(X)$, $N'(X') = N(X)$, $\theta'(X') = \theta(X)$
- *Corollary* : Assume $\varphi'_{X'} = \varphi_X$, $X \in \Phi$ (possibly satisfying the load equations). Then $\mathbf{E}'^0[\rho'(0)] = \mathbf{E}^0[\rho(0)]$ and $\mathbf{E}'^0[\theta'(0)] = \mathbf{E}^0[\theta(0)]$. Consequently, the mean cells characteristics associated to Φ and Φ' are identical

Inhomogeneous networks with homogeneous QoS response

- A country is composed of urban, suburban and rural areas
  - The parameters K and β of the deterministic part of propagation loss depend on the type of the zone.
  - Assume that for each zone i

$$K_i/\sqrt{\lambda_i} = \mathrm{const}$$

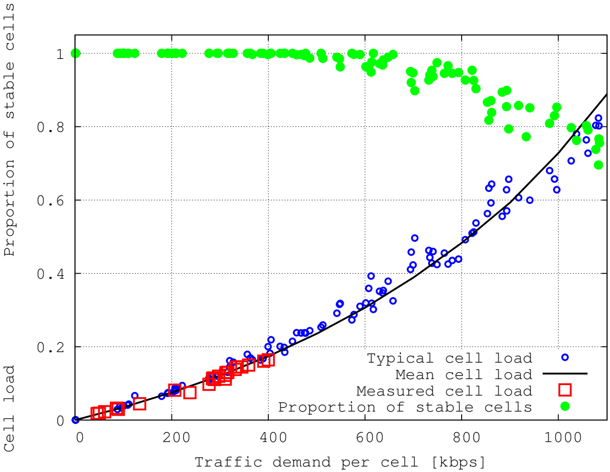
- Then the scaling laws say that
  - locally, for each homogeneous area of this inhomogeneous network, one will observe the same relation between the mean performance metrics and the (per-cell) traffic demand
  - In other words, one relation is enough to capture the key dependence between performance and traffic demand for different areas of this network

Numerical setting

- 3G network at carrier frequency $f_0 = 2.1$GHz with frequency bandwidth $W = 5$MHz
- Distance-loss function $l(r) = (Kr)^{\beta}$, with $K = 7117$km^{-1}, $\beta = 3.8$ (COST Walfisch-Ikegami model [4])
- Log-normal shadowing with standard deviation 9.6dB and the mean spatial correlation distance 100m
- Transmision power is $P = 60$dBm, with fraction $\epsilon = 0.1$ for pilot channel, noise power $N = -96$dBm
- 3D antenna pattern specified in [5, Table A.2.1.1-2]
- $R\,(\mathrm{SINR}) = 0.3 \times W\mathbf{E}\left[\log_2\left(1 + |H|^2\,\mathrm{SINR}\right)\right]$ where H Rayleigh fading satisfying $\mathbf{E}[|H|^2] = 1$
- Poisson process of BS with intensity $\lambda = 1.27$km^{-2} (average cell radius 0.5km) within a disc of radius 5km

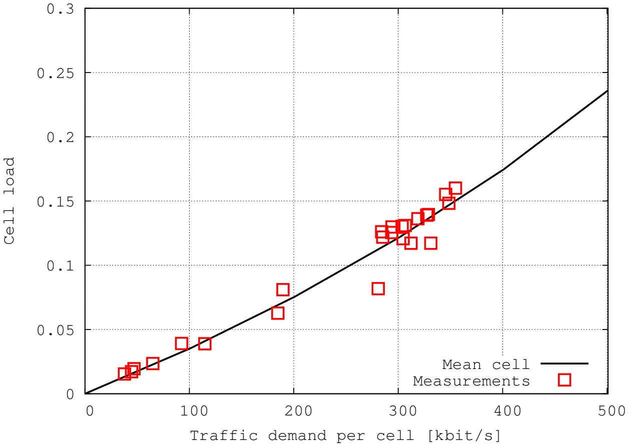
European city

- Typical cell and mean cell models predict similar values of the average load



Large region in an European country

- comprizing urban, suburban and rural areas



Conclusion

- Two approaches based on stochastic geometry in conjunction with queueing and information theory are developed
  - In order to evaluate performance metrics in large irregular cellular networks
  - Typical cell approach : spatial averages
  - Mean cell approach : simpler, approximate but fully analytic
- We validate the proposed approach by showing that it allows to predict the performance of a real network
- Further work
  - Spatial distribution of the performance metrics [6]
  - For multi-tier networks, calculate the characteristics of each tier [7]

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