Estimating the Transmission Probability in Wireless Networks with Configuration Models

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Context

- We consider a large set of nodes which communicate with each other by means of a wireless channel.
- The medium access control (MAC) is defined as a CSMA-like protocol where the RTS/CTS handshake is used to avoid the hidden node problem.
- Time divide in slots.
- Other assumptions:
  - transmissions are perfectly received or not received at all
  - symmetric channel: similar hardware and same transmission power
  - RTS/CTS handshake takes place instantaneously
  - saturated traffic

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We are interested in the transmission probability, that is the number of concurrent successful transmissions (eq. spatial reuse)

Main Contribution

- We propose a new methodology to estimate the transmission probability in this context.
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- We propose a new methodology to estimate the transmission probability in this context.
- We model the interferences between users as a random graph.
- Using configuration models for random graphs, we show how the properties of the medium access mechanism are captured by some deterministic differential equations, when the size of the graph gets large.
- Performance indicators such as the transmission probability can then be efficiently computed from these equations.
- We show that our results are accurate for different types of random graphs and that even on spatial structures, these estimates get very accurate as soon as the variance of the interference is not negligible.

CSMA Scheduling

At time 0, every node will choose a random number, uniformly distributed between 0 and T_c (backoff time):

- the node with the minimum such time will send a RTS frame to one of its neighbors, chosen randomly among them. All the overhearing neighbors will be blocked.
- if the destination node answer with a CTS frame, then it will also block all its neighbors.
- the origin node will immediately start transmitting a data frame

We will term these two nodes as **active** ($\mathcal{A}$), their neighbors as **blocked** ($\mathcal{B}$) and the rest of the nodes as **unexplored** ($\mathcal{U}$).

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- the procedure is repeated until time T_c and the proportion of actives nodes at the end of the procedure is the transmission probability.

Parking Process

- The network can be described as an interference graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ and the CSMA schedule as a parking process on $\mathcal{G}$.
- Assume that each node have an independent exponential clock. At time 0 we have $\mathcal{A}_0 = \mathcal{B}_0 = \emptyset$ and $\mathcal{U}_0 = \mathcal{V} = \{1, \ldots, n\}$.
  1. at time t_1 the minimum of these N competing exponential the transmitting node s is uniformly chosen from $\mathcal{U}_{t_1^-}$
  2. a random unexplored neighbor r of s is chosen (if any), and *if it is able to answer with a CTS*, then all the unexplored neighbors of r ($\mathcal{N}_r$) and s ($\mathcal{N}_s$) are blocked, and we further update the sets as:

$$\mathcal{A}_{t_1^+} \leftarrow \mathcal{A}_{t_1^+} \cup \{s, r\},$$
$$\mathcal{B}_{t_1^+} \leftarrow \mathcal{B}_{t_1} \cup \mathcal{N}_s \cup \mathcal{N}_r,$$
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- The number of active nodes is the jamming constant of the graph
- Remark: we analyze different ways to chose the receiver and also different ways to react to a CTS failure

Configuration Model $CM_n(d)$

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- How we construct the graph? given an n-sample of the degree distribution, the half-edges are linked uniformly:
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- Key feature: the number of self-loops and multiple edges are negligible when the number of nodes tends to infinity

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- At time t, for a given unexplored node $i \in \mathcal{U}_t$ consider $d_i(\mathcal{U}_t)$ the degree of i toward $\mathcal{U}_t$, *i.e.*

  $d_i(\mathcal{U}_t) =$ number of half-edges emanaiting from i and pointing to $\mathcal{U}_t$

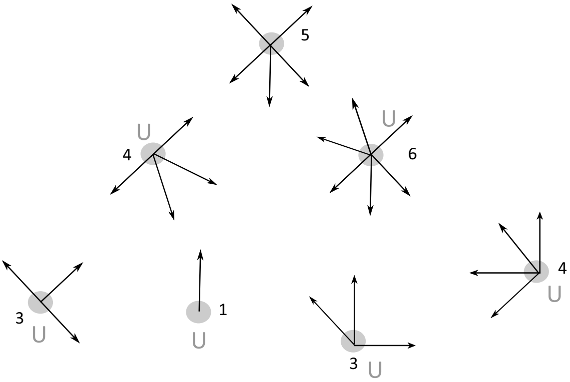
- Empirical degree distribution of the unexplored nodes

$$\mu_t = \sum_{i \in \mathcal{U}_t} \delta_{d_i(\mathcal{U}_t)}.$$

- Then $\{\mu_t\}_{t \geq 0}$ is a measure-valued Markov Process.

Evolution of μ_t - Step 1

At the beginning...

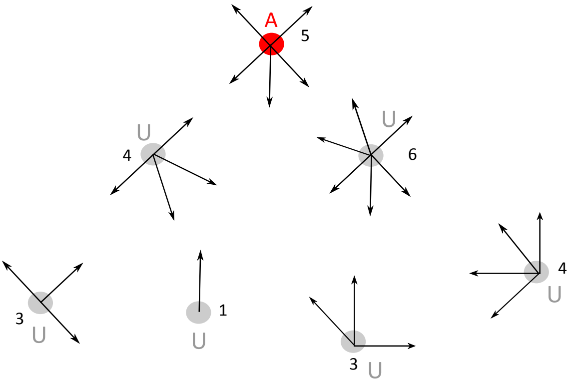


$$\mu_{t^-} = \delta_1 + 2\delta_3 + 2\delta_4 + \delta_5 + \delta_6$$

so that the associated graph has $n = \langle \mu_{t^-}, \mathbf{1} \rangle = 7$ unexplored nodes and $\langle \mu_{t^-}, \chi \rangle = 26$ half-edges

Evolution of μ_t - Step 1

A clock rings at t: the new A-node has degree $K(\mu_{t^-}) = 5$

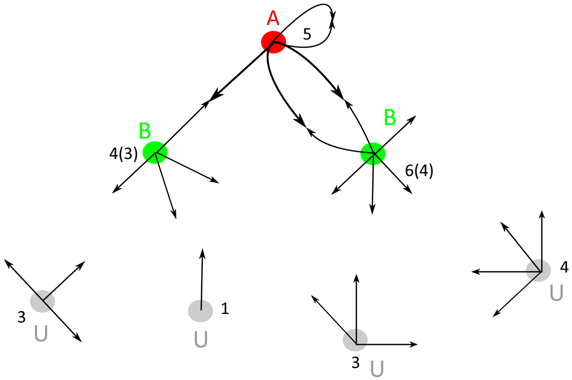


Measure before the transition:

$$\mu_{t^-} = \delta_1 + 2\delta_3 + 2\delta_4 + \delta_5 + \delta_6,$$
$$\mu_{t^{-+}} = \delta_1 + 2\delta_3 + 2\delta_4 + \delta_6.$$

Evolution of μ_t - Step 2

Half-edges are matched with another one drawn uniformly at random

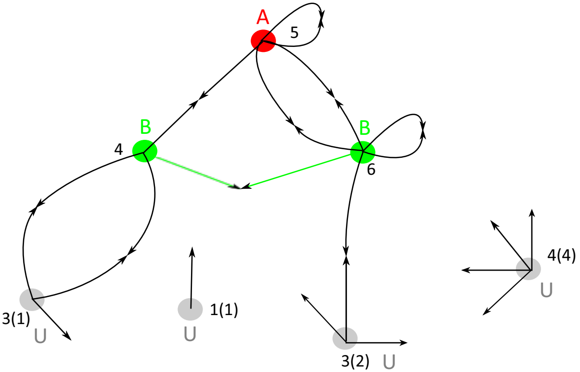


Measure update:

$$\mu_{t^{-+}} = \delta_1 + 2\delta_3 + 2\delta_4 + \delta_6,$$
$$\mu_{t^{-+}} = \delta_1 + 2\delta_3 + \delta_4.$$

Evolution of μ_t - Step 3

We repeat the pairing with the neighbors of the selected node

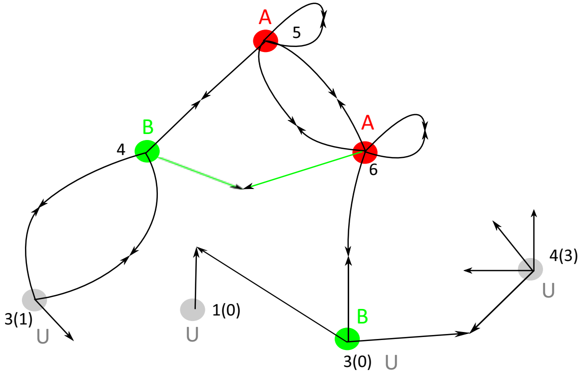


Measure update:

$$\mu_{t^{-+}} = \delta_1 + 2\delta_3 + \delta_4,$$
$$\mu_{t^{-+}} = 2\delta_1 + \delta_2 + \delta_4$$

Evolution of μ_t - Step 4

An unexplored neighbor is chosen at random as receiver



Measure after the transition

$$\mu_{t^{-+}} = 2\delta_1 + \delta_2 + \delta_4,$$
$$\mu_t = \delta_1 + \delta_3$$

Large Graph Limit

For all size n, define the scaled measure

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Under suitable initial assumptions, for all T and all bounded test ϕ,

$$\sup_{t\in[0,T]} |\langle \bar{\mu}_t^n, \phi\rangle - \langle \bar{\mu}_t, \phi\rangle| \xrightarrow[n\to\infty]{(\mathcal{P})} 0,$$

where $\bar{\mu}$ is the unique solution of an infinite differential equation system.

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where $\bar{\mu}$ is the unique solution of an infinite differential equation system.

- The proof is based in classical martingale decomposition results for Markov processes
- The uniqueness of the deterministic limiting measure is proved using an adequate norm on the spaces of solutions.

Fluid Differential Equation System

In particular for $\phi = \delta_i$, the ordinary differential equation is

$$\frac{d\bar{\mu}_t(i)}{dt} = F_t(i)\,(\bar{\mu})\,,$$

where the drift $F_t(i)$ is the mean number of nodes with i half-edges of type $\mathrm{U} \to \mathrm{U}$ that are removed at t if a transition occurs, times the normalized transition rate.

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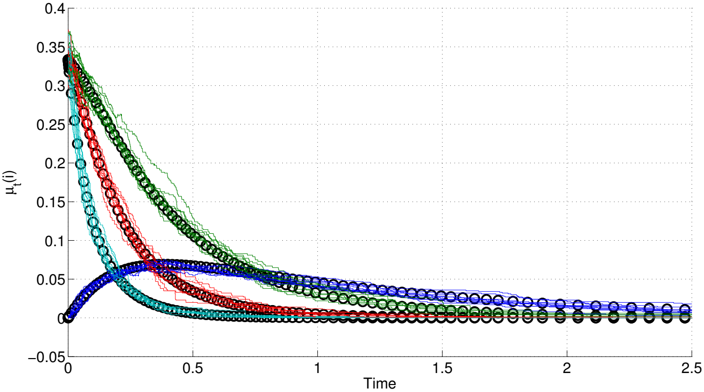
$F_t(i)$ depends on the distributions $\bar{\alpha}_t$ and $\bar{\beta}_t$:

- $\bar{\alpha}_t(i) = \frac{\bar{\mu}_t(i)}{\langle \mu_t, 1 \rangle}$ degree distribution of a randomly picked unexplored node at time t,
- $\bar{\beta}_t(i) = \frac{i\bar{\mu}_t(i)}{\langle \mu_t, \chi \rangle}$ size biased distribution of α_t, degree distribution of any neighbor of a randomly picked unexplored node.

Approximation for a Finite Number of Nodes



An example comparing several realizations of μ_t^N ($N = 1000$) and the solution of the previous equation (marked as circles), where $\mu_0^N = \bar{\mu}_0 = (0, 1/3, 1/3, 1/3)$.

Jamming Constant - Spatial Reuse

- Let A_t^n be the number of active nodes ($\mathcal{A}$) at time t, the *Jamming Constant* $\bar{J}^n$ then reads

$$\bar{J}^n = \lim_{t\to\infty} \frac{A_t^n}{n} \text{ a.s..}$$

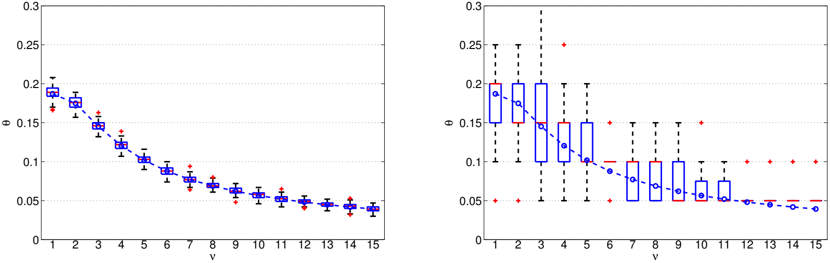
Under the assumptions of our main result, we obtain that

$$\bar{J}^n \underset{n\to\infty}{\longrightarrow} c_{\bar{\mu}_0} = \lambda \int_0^\infty \sum_{j>0} \bar{\mu}_t(j)dt = \lambda \int_0^\infty \bar{u}(t)P_t(CTS)dt.$$

- $c_{\bar{\mu}_0}$ is an explicit formula for the spatial reuse that can be easily calculated from the solution of the differential equation system.

Reference: “The Jamming Constant of Uniform Random Graph” P.Bermolen, M.Jonckheere and P.Moyal in arXiv and submitted.

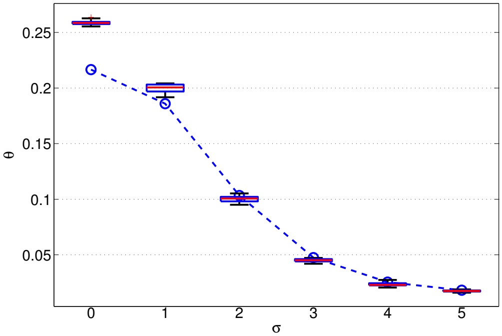
Example 1: Spatial Reuse for a Poisson Distribution



Evaluation of differntial equation along with the boxplot of the numerical results of 100 simulations for $N = 1000$ (left) and $N = 20$ (right). The initial nodes' degree is distributed as a Poisson with parameter ν.

Example 2: Spatial Reuse for Spatial Models

Poisson process with log-normal fading and a path-loss of the form $L(r) = r^{-2}$



Evaluation of the differential equation along with the boxplot of the numerical results of 10 time-slot simulations. The value of σ corresponds to the standard deviation of the underlying normal distribution.

Model Extensions

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   $\Rightarrow$ this scenario can be analyzed with a bi-dimensional measure $\mu_t(i, j)$ that represents the number of nodes with i half-edges toward the unexplored set and j half-edges toward de the blocked set.

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   $\Rightarrow$ this scenario can also be analyzed by defining a new class of nodes (only receivers) and two coupled multidimensional measures.

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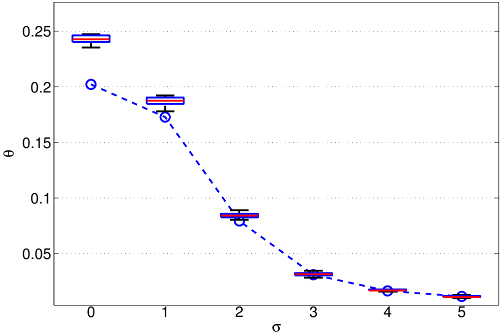
The simulation results are qualitatevily the same we presented here.

Example of Model Extension 2

Poisson process with log-normal fading and a path-loss of the form $L(r) = r^{-2}$



Evaluation of the corresponding differential equation system along with the boxplot of the numerical results of 10 time-slot simulations.

¡MUCHAS GRACIAS!