

An Analytic Study of Climate Sensitivity

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Abstract

According to IPCC, the climate sensitivity in the absence of feedbacks is $\Delta T = 1^\circ\text{C}$. The evaluation is based on the Stefan-Boltzmann law. This naive picture is however doubtful. In the present work we have examined the climate sensitivity in the analytic model of radiative transfer and found that the IPCC is incorrect. The precise climate sensitivity is $\Delta T = 1.4^\circ\text{C}$. The observed temperature anomaly is approximately reproduced even in the absence of feedbacks. It is likely that the feedbacks are rather weak. So as to confirm this speculation, we have calculated the climate sensitivity with the water vapor feedback which is however obviously overestimated. The resultant feedback factor 1.65 is lower than 2 predicted by IPCC. In addition, we artificially reduce the water vapor feedback by half. The result reproduces the observed temperature anomaly fairly well. The resultant feedback factor 1.25 is much lower than 2. We can therefore conclude that the positive feedbacks are much weaker than the IPCC predictions.

1 Introduction

Now it is circulated [1-5] that the anthropogenic global warming (AGW), which is mostly due to the increase in carbon dioxide (CO_2) concentration from fossil fuel burning, is a matter of life and death to humanity. Such a sense of impending crisis is outstanding in a catchword “Dangerous Anthropogenic Interference” (DAI) with the climate system, which was first stated in Article 2 [6] of the United Nations Framework Convention on Climate Change. It has been widely endorsed since the European Union pronounced that the increase in average global temperature should be below 2°C [7-10] above pre-industrial temperature so as to avert the irreversible climate catastrophe. At present, some scientists [11-15] warn further that the present global warming has already exceeded the DAI threshold of 2°C because the cooling due to aerosols masks the warming due to greenhouse gases (GHGs). They insist [16] that the present CO_2 concentration of 390ppm must be decreased below 350ppm. However, the American Chemical Society explicitly states [17] that aerosols cannot offset much of the warming from GHGs because of their short lifetimes. Moreover, as pointed out in Ref. [18] the climate sensitivity estimated in the Fourth Assessment Report (AR4) [19] of Intergovernmental Panel on Climate Change (IPCC) might be too high. These suggest that the IPCC School overestimates both the positive and negative forcing. In the present work, we investigate this problem.

2 Climate sensitivity without feedbacks

We first consider the radiative forcing in the absence of feedbacks. In this case, the physics of IPCC School is naive. It is based on the Stefan-Boltzmann law [20,21]:

$$\lambda_0 = \frac{T_S}{4\sigma T_E^4}, \quad (1)$$

where σ is the Stefan-Boltzmann constant, $T_E = 255\text{K}$ and $T_S = 288\text{K}$. The naive picture of Eq. (1) is sharply contrast to the elaborate and enormous supercomputer simulations in AR4 [19]. This is really surprising and raises doubts about the climate sensitivity predicted by IPCC. So as to assess the reliability of IPCC, we have to overcome the naive picture of Eq. (1). An analytic model of radiative transfer is necessary. The present author developed such a model. It is simple but useful as shown in Ref. [22].

According to IPCC, which states [19] that the water vapor does not contribute to radiative forcing directly but plays a role in feedback, the radiative forcing in our model is given by

$$-\lambda_0 = \frac{dT}{dF} = \left(\frac{dT}{d\tau_C} \right) / \left(\frac{dF}{d\tau_C} \right), \quad (2)$$

where τ_C is the optical depth of atmospheric CO_2 . T is the surface air temperature given by

$$T^4 = \frac{2 + 3\tau}{4 + 3(1 - \beta)\tau} T_E^4, \quad (3)$$

where $1 - \beta = 0.3$ is a fraction of the thermal window between the wave number of 800cm^{-1} and 1250cm^{-1} to the total thermal radiation from surface. (In our model [22] we only take into account CO_2 and water vapor as GHGs but does not O_3 .) The atmospheric total optical depth τ is given by

$$\frac{1}{\tau} = \frac{\beta_C}{\beta} \frac{1}{\tau_{CW}} + \frac{\beta_W}{\beta} \frac{1}{\tau_W}, \quad (4)$$

where $\beta_C = 0.2$ is a fraction of CO_2 absorption band between the wave number of 600cm^{-1} and 800cm^{-1} to the total thermal radiation, $\beta_W = \beta - \beta_C = 0.5$, τ_W is the optical depth of water vapor and $\tau_{CW} = \tau_C + \tau_W$ is the optical depth of CO_2 absorption band.

On the other hand, F in Eq. (2) is the upwelling flux from CO_2 absorption band at the top of atmosphere (TOA):

$$F = 2\beta_C \frac{1 + \tau/\tau_{CW}}{4 + 3(1 - \beta)\tau} \sigma T_E^4. \quad (5)$$

Using Eq. (4) this is rewritten as

$$F = 2 \frac{(\beta + \beta_C) + \beta_W \left(\tau / \tau_W^{(0)} \right)}{4 + 3(1 - \beta) \tau} \sigma T_E^4. \quad (6)$$

Because the water vapor does not contribute to radiative forcing, the optical depth τ_W has been fixed to its pre-industrial value of $\tau_W^{(0)}$. Therefore, the differentiations in terms of τ_C in Eq. (2) can be replaced by the differentiations in terms of τ . From Eqs. (3) and (6) we have

$$\frac{dT}{d\tau} = \frac{6(1 + \beta)}{[4 + 3(1 - \beta)\tau]^2} \frac{T_E^4}{4T^3}, \quad (7)$$

$$\frac{dF}{d\tau} = -2 \frac{4\beta_W + 3(1 - \beta)(\beta + \beta_C)\tau_W^{(0)}}{[4 + 3(1 - \beta)\tau]^2 \tau_W^{(0)}} \sigma T_E^4. \quad (8)$$

Consequently, the radiative forcing in the absence of feedbacks is given by

$$\lambda_0 = f \frac{T_S}{4\sigma T_S^4}, \quad (9)$$

$$f = \frac{3(1 + \beta)\tau_W^{(0)}}{4\beta_W + 3(1 - \beta)(\beta + \beta_C)\tau_W^{(0)}}. \quad (10)$$

At end of the calculations we have fixed the temperature T to its pre-industrial value T_S .

It is noted that T_E^4 on the denominator in the right hand side of Eq. (1) has been replaced by T_S^4 in Eq. (9). This is first pointed out in Ref. [23] and shows a serious drawback in the physics of IPCC School. Moreover, note the negative sign in the left hand side of Eq. (2). It is necessary because the basic physical picture of AGW is that the global warming is caused by decrease in the outgoing long-wave radiation (OLR) from the Earth due to increase in GHGs. On the other hand λ_0 must be positive. Therefore, the right hand side of Eq. (2) must be negative. In our model this requirement is naturally satisfied because of the negative sign in the right hand side of Eq. (8). To the contrary, the IPCC is forced to compensate the negative sign in the left hand side of Eq. (2) by hand because the Stefan-Boltzmann law never produces the negative sign in contrast to Eq. (8).

So as to calculate the climate sensitivity, we have to determine the pre- and post-industrial values of optical depths. First, the atmospheric total optical depth $\tau^{(0)}$ in pre-industrial era is calculated from Eq. (3) as follows:

$$\tau^{(0)} = \frac{2}{3} \frac{2T_S^4 - T_E^4}{T_E^4 - (1 - \beta)T_S^4} = 2.94. \quad (11)$$

The water vapor optical depth is evaluated as follows. According to Ref. [24], CO₂ contributes to the atmospheric greenhouse effect by 20%. Because in our model [22] the

greenhouse effect is only due to water vapor and CO₂, this means that the water vapor contributes to the greenhouse effect by 80%. Therefore, replacing T_S in Eq. (11) by $T_W = T_E + 0.8 \times (T_S - T_E)$ we determine the water vapor optical depth $\tau_W^{(0)}$ in pre-industrial era:

$$\tau_W^{(0)} = \frac{2}{3} \frac{2T_W^4 - T_E^4}{T_E^4 - (1 - \beta)T_W^4} = 2.36. \quad (12)$$

Using Eqs. (11) and (12), the optical depth of CO₂ absorption band in pre-industrial era is determined from Eq. (4). Then, the pre-industrial value of τ_C is given by $\tau_C^{(0)} = \tau_{CW}^{(0)} - \tau_W^{(0)}$. On the other hand, we assume that the post-industrial value of τ_C is given by

$$\tau_C(n) = \frac{n}{n_0} \tau_C(n_0) = \frac{n}{n_0} \tau_C^{(0)}, \quad (13)$$

where $n_0 = 280$ ppm and n are the pre- and post-industrial values of CO₂ concentration, respectively. Then, the post-industrial value of τ_{CW} is given by $\tau_{CW}(n) = \tau_C(n) + \tau_W(n_0)$. It is noted again that the optical depth τ_W is fixed to its pre-industrial value $\tau_W(n_0) = \tau_W^{(0)}$ because the water vapor does not contribute to radiative forcing directly. Substituting $\tau_{CW}(n)$ and $\tau_W(n_0)$ into Eq. (4), we calculate the total optical depth of $\tau(n)$ in post-industrial era.

Using $\tau(n)$, $\tau(n_0) = \tau^{(0)}$ and $\tau_W(n_0) = \tau_W^{(0)}$, the decrease in OLR is calculated from Eq. (6):

$$\begin{aligned} \Delta F(n) &= F(n) - F(n_0) \\ &= 2 \left\{ \frac{(\beta + \beta_C) - \beta_W \left[\tau(n) / \tau_W^{(0)} \right]}{4 + 3(1 - \beta) \tau(n)} - \frac{(\beta + \beta_C) - \beta_W \left[\tau(n_0) / \tau_W^{(0)} \right]}{4 + 3(1 - \beta) \tau(n_0)} \right\} \sigma T_E^4. \end{aligned} \quad (14)$$

Finally, according to the IPCC School, we calculate the temperature anomaly in post-industrial era as follows:

$$\Delta T(n) = \lambda_0 \times \Delta F(n) \quad (15)$$

The climate sensitivity is given by $\Delta T(n = 2n_0)$.

Our numerical results are largely different from the IPCC predictions. Although $\Delta F = 2.47 \text{ W/m}^2$ is lower than the IPCC prediction $\Delta F = 5.35 \times \ln(2) = 3.71 \text{ W/m}^2$ [25], the climate sensitivity of $\Delta T = 1.4 \text{ }^\circ\text{C}$ is higher than the IPCC prediction $\Delta T = 1 \text{ }^\circ\text{C}$ [5]. The difference is due to the factor $f = 3.08$ that is absent in the theory of IPCC. Although the IPCC School insists [5] that the climate sensitivity of $\Delta T = 1 \text{ }^\circ\text{C}$ in the absence of feedbacks is widely agreed, the value is doubtful.

The green curve in Fig. 1 shows the temperature anomaly after 1850. For the calculation we use the data sets of CO₂ concentration from Carbon Dioxide Information Analysis Center (CDIAC) [26] and National Atmospheric Administration (NOAA) [27].

It is noted that the observed global warming can be approximately reproduced even in the absence of feedbacks. This suggests that the positive feedbacks are much weaker than the IPCC predictions.

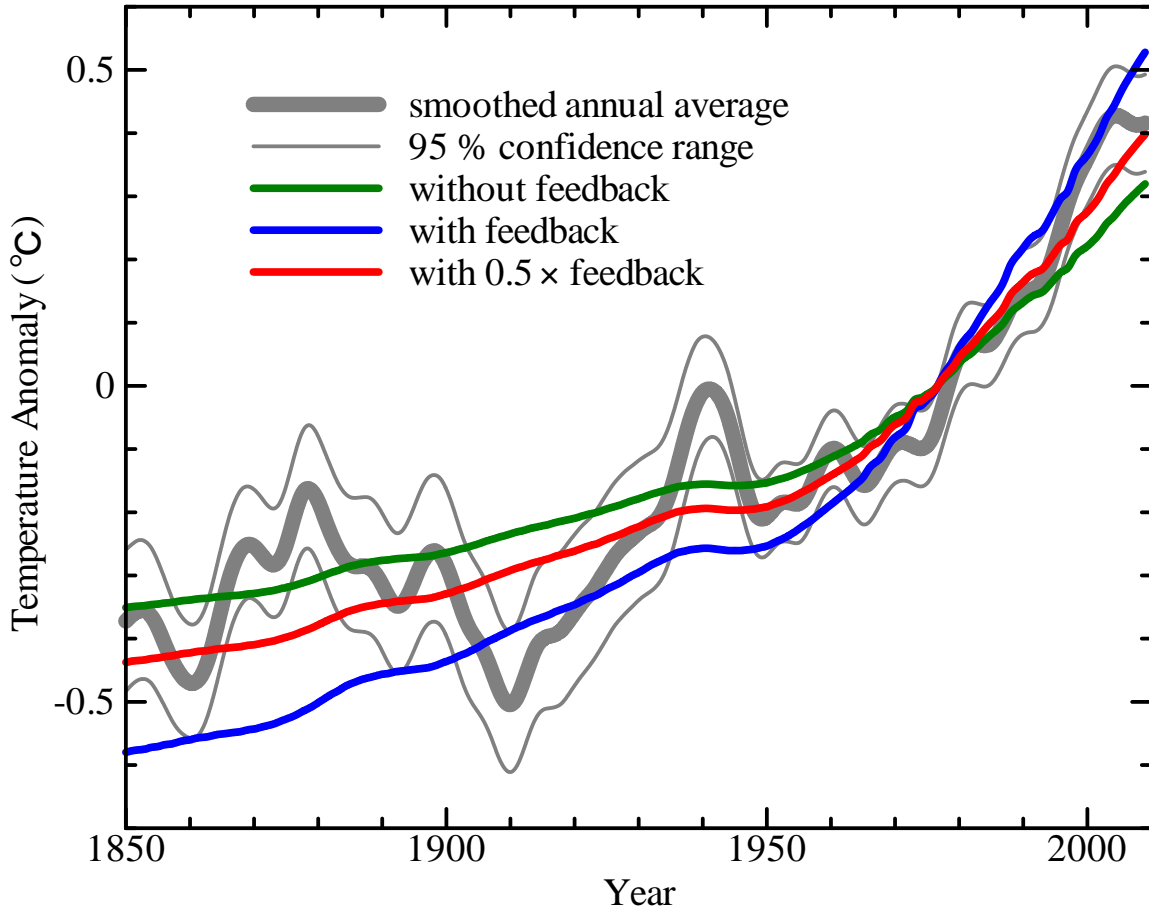


Figure 1: The gray curves are the observed temperature anomalies in Met Office Hadley Centre observations datasets [28]. The blue and green curves are the model calculations with and without water vapor feedback, respectively. The red curve is calculated in reducing the feedback by half.

3 Climate sensitivity with water vapor feedback

The IPCC School predicts [5] that the climate sensitivity is $\Delta T = 1^\circ\text{C}$ in the absence of feedbacks. On the other hand, the IPCC School predicts [5] that the water vapor feedback approximately doubles it. Therefore, we have the climate sensitivity of $\Delta T = 2^\circ\text{C}$ with water vapor feedback. Because the value is just the DAI threshold, the water vapor feedback is crucial in the warning to AGW. The value is also just the lower bound of the overall climate sensitivity predicted in AR4 [19], which concludes that the overall climate sensitivity is likely to lie between 2°C and 4.5°C with a most likely value of approximately

3°C. The overall climate sensitivity over 2°C is due to the feedbacks other than water vapor. It is believed [5,29] that the cloud feedback is the most important one among them.

However, as shown in the preceding section, the IPCC prediction of $\Delta T = 1^\circ\text{C}$ is doubtful. If our prediction $\Delta T = 1.4^\circ\text{C}$ is assumed and if the water vapor feedback approximately doubles it, we have $\Delta T = 2.8^\circ\text{C}$. The value agrees with the most likely value of the overall climate sensitivity predicted in Ref. [30] and is similar to $\Delta T = 3^\circ\text{C}$ in AR4. This indicates that the feedbacks other than water vapor are much weaker than the IPCC predictions. On the other hand, if the climate sensitivity with water vapor feedback alone should be $\Delta T = 2^\circ\text{C}$, the water vapor feedback is much weaker than the IPCC prediction. In the following we will explicitly verify that the IPCC overestimates the feedbacks.

According to Fig. 6a in Ref. [2], the mechanism of water vapor feedback is as follows. First, the increase in CO_2 causes warming. This is expressed in terms of $\partial T/\partial \tau_C$. Next, the resultant warming leads to more water vapor in atmosphere. This is expressed in terms of $d\tau_W/dT$. Finally, the resultant more water vapor leads to the enhanced greenhouse effect. This is expressed in terms of $\partial F/\partial \tau_W$. Therefore, we can take into account water vapor feedback by means of the following prescription in the right hand side of Eq. (2):

$$\frac{dF}{d\tau_C} \rightarrow \frac{\partial F}{\partial \tau_C} + \frac{\partial F}{\partial \tau_W} \frac{d\tau_W}{dT} \frac{\partial T}{\partial \tau_C}. \quad (16)$$

Consequently, the radiative forcing is given by

$$\lambda = \frac{dT}{d\tau} \left(\frac{dF}{d\tau} + \frac{\partial F}{\partial \tau_W} \frac{d\tau_W}{dT} \frac{dT}{d\tau} \right)^{-1}. \quad (17)$$

As seen in Eq. (8), $dF/d\tau$ is negative. Therefore, if we perform a replacement $dF/d\tau \rightarrow -dF/d\tau$, Eq. (17) can be expressed in a general form [31]:

$$\lambda = \frac{\lambda_0}{1 - C \lambda_0}, \quad (18)$$

where

$$C = \frac{\partial F}{\partial \tau_W} \frac{d\tau_W}{dT}. \quad (19)$$

Here, it is noted that $\partial T/\partial \tau_C$ in Eq. (16) is calculated from Eq. (3) but $d\tau_W/dT$ is not equivalent to $(\partial T/\partial \tau_W)^{-1}$ calculated from Eq. (3). We cannot replace Eq. (19) by

$$C = \left(\frac{\partial F}{\partial \tau_W} \right) / \left(\frac{\partial T}{\partial \tau_W} \right). \quad (20)$$

If we use Eq. (20), the water vapor feedback is overestimated because $\partial T/\partial \tau_W$ attributes

the temperature rise entirely to the increase in water vapor and because Eq. (20) does not contain the negative feedback effect due to latent heat.

Nevertheless, Eq. (20) is useful. If the overestimated results are similar to the IPCC predictions, we see that the IPCC School overestimates AGW. Therefore, we go ahead and use Eq. (20) in place of Eq. (19) anyway. As a result, we have an analytic expression of $C \lambda_0$:

$$C \lambda_0 = \frac{4\beta/\tau_C^{(0)} + 3(1-\beta) \left[\beta \left(\tau^{(0)}/\tau_C^{(0)} \right) + \beta_W \left(\tau^{(0)}/\tau_W^{(0)} \right) - (\beta + \beta_C) \right]}{4\beta_W + 3(1-\beta)(\beta + \beta_C) \tau_W^{(0)}} \tau_W^{(0)}. \quad (21)$$

At end of the calculation we have fixed the optical depths to their pre-industrial values.

Now, we can readily calculate the temperature anomaly with water vapor feedback:

$$\Delta T(n) = \lambda \times \Delta F(n), \quad (22)$$

where ΔF is the same as Eq. (14). The numerical result is shown by the blue curve in Fig. 1. Although the rapid warming after 1950 is well reproduced, the calculation is much lower than observation before 1950. Because Eq. (21) overestimates the water vapor feedback, the result is as expected. Therefore, the resultant climate sensitivity of $\Delta T = 2.32^\circ\text{C}$ is also overestimated. However, the feedback factor $2.32/1.4 = 1.65$ is lower than 2 predicted by IPCC School. Consequently, we see that the IPCC School overestimates the water vapor feedback.

Moreover, as an experiment, we attempt to calculate the temperature anomaly using the water vapor feedback reduced artificially by half:

$$\lambda = \frac{\lambda_0}{1 - \frac{1}{2} C \lambda_0}. \quad (23)$$

The result is shown by the red curve in Fig. 1. It is able to reproduce the observation fairly well. The resultant climate sensitivity of $\Delta T = 1.75^\circ\text{C}$ is therefore reliable. Because the water vapor feedback factor $1.75/1.4 = 1.25$ is much lower than 2 predicted by IPCC School, it is concluded that the IPCC School largely overestimates the water vapor feedback.

In our model, we only consider water vapor and CO_2 as GHGs, and it is assumed that the water vapor contributes to the atmospheric greenhouse effects by 80%. To the contrary, according to Ref. [24] the contribution of water vapor is 50% and the greenhouse effect by cloud amounts to 25%. According to Ref. [5] the overall climate sensitivity above 2°C is largely due to the feedback from clouds. Do these mean that our result of $\Delta T = 1.75^\circ\text{C}$ underestimates the climate sensitivity because of neglecting clouds? No, their effects are implicitly included in the greenhouse effect of water vapor. Because our water vapor feedback factor 1.25 agrees with the cloud feedback factor in Ref. [33], our model essentially takes into account the cloud feedback. This is the reason

for the success of the red curve in Fig. 1. To the contrary, the IPCC School probably overestimates the feedbacks other than water vapor.

The recent analysis [34] suggests that the rapid warming after 1980 is the result from the naturally caused climate oscillation of 60-years cycle. Because our model does not take into account natural forcing, it is not a problem that the green curve in Fig. 1 cannot reproduce the observation after 1990. Therefore, the observed temperature anomaly never excludes the result with no feedbacks. In fact, the climate sensitivity of $\Delta T = 1.4^\circ\text{C}$ is allowed in the recent study [32]. At present, we cannot prefer the red curve to the green curve in Fig. 1. This indicates that the IPCC School overestimates not only the water vapor feedback but also the other positive feedbacks. Although the IPCC School stresses that the strong positive feedbacks are masked by the cooling due to aerosols, the overestimates of positive feedbacks mean that the IPCC School also overestimates the negative forcing by aerosols.

4 Conclusion

Based on the Stefan-Boltzmann law, the IPCC derives the climate sensitivity of $\Delta T = 1^\circ\text{C}$ in the absence of feedbacks. Is this naive picture reasonable? In the present work we examine the radiative forcing in a refined theoretical framework based on an analytic model of radiative transfer. We have found that the naive picture of IPCC is incorrect. The precise climate sensitivity is $\Delta T = 1.4^\circ\text{C}$. The observed temperature anomaly can be reproduced even in the absence of feedbacks. The result is quite suggestive. Although the IPCC derives the overall climate sensitivity of $\Delta T = 3^\circ\text{C}$, the value might be too high as pointed out in Ref. [18]. So as to assess the overestimates by IPCC, we examine the climate sensitivity using a pure analytic expression of water vapor feedback, which is however expected to produce the results overestimated. The obtained value $\Delta T = 2.32^\circ\text{C}$ is similar to the overall climate sensitivity predicted in Ref. [32]. The water vapor feedback factor 1.65 is however lower than 2 predicted by IPCC. It is therefore seen that the IPCC overestimates the water vapor feedback. In addition, as an experiment, we continue the calculation using the water vapor feedback reduced artificially by half. The result can reproduce the observed temperature anomaly fairly well. The resultant feedback factor 1.25 agrees with the cloud feedback factor in Ref. [33] but is much weaker than the IPCC prediction. This indicates that our model effectively includes the cloud feedback and that the IPCC also overestimates the positive feedbacks other than water vapor. Moreover, the overestimates of positive feedbacks also indicate that the IPCC overestimates the negative forcing by aerosols. Consequently, we can say that the IPCC exaggerates the anthropogenic effects on climate.

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