UNIVERSITY OF TEXAS AT AUSTIN

# HW Assignment 3

**Problem 3.1.** For a utility function $U : \mathbb{R} \to \mathbb{R}$ with range $\mathbb{R}$, we define the **certainty equivalent** $c(U, X) \in \mathbb{R}$ of the random variable $X$ with $U(X) \in \mathbb{L}^1$, as the (unique) solution to the following, **indifference**, equation

$$U(c(U, X)) = \mathbb{E}[U(X)].$$

A utility function $U : \mathbb{R} \to \mathbb{R}$ is said to exhibit **decreasing absolute risk aversion** if the function $r_U$ is strictly decreasing. Set $\mathcal{X} = \{X : U(x + X) \in \mathbb{L}^1, \forall x \in \mathbb{R}\}$. Show that the following are equivalent for $U : \mathbb{R} \to \mathbb{R}$ with range $\mathbb{R}$:

(1) $U$ exhibits decreasing relative risk aversion,
(2) the function $x - c(U, x + X)$ is decreasing in $x$, for each $X \in \mathcal{X}$
(3) for all $x_1 < x_2 \in \mathbb{R}$ there exists a concave function $\psi : \mathbb{R} \to \mathbb{R}$ such that $u(x_1 + z) = \psi(u(x_2 + z))$.

(Note: assume enough differentiability, if you want to make mathematics simpler.)

**Problem 3.2.** Find an example of a preference relation that does not admit an expected-utility representation.

**Problem 3.3.** Suppose that $\preceq$ is a preference relation on the set $\mathcal{X}$ of all random variables on $\Omega$ which admits an expected-utility representation. Show that it satisfies the following property (called the **sure-thing principle**):

*For any choice of $X_1, X_2, \hat{X}_1, \hat{X}_2 \in \mathcal{X}$ and $A \subseteq \Omega$ such that*

- *$X_1 = X_2$ and $\hat{X}_1 = \hat{X}_2$ on $A$ and*
- *$X_1 = \hat{X}_1$ and $X_2 = \hat{X}_2$ on $A^c$,*

*we have*

$$X_1 \preceq X_2 \Leftrightarrow \hat{X}_1 \preceq \hat{X}_2.$$

(Note: It can be shown that a converse holds under certain, additional, regularity assumptions: preference+sure-thing $\Rightarrow$ expected utility.)