rtg2010
August 11 - Challenges

1. Consider the Hénon map

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$
$$f(x, y) = \left(1 - ax^2 + y, bx\right)$$

with classical parameters $a = 1.4$ and $b = 0.3$.

   (a) Using standard numerical tools of your choice find some periodic orbits of periods two, three, four and five.

   (b) Using interval arithmetic and the interval Newton method verify that there truly exist periodic orbits inside given neighbourhoods of your numerical guesses.

2. Consider the planar restricted three body problem

$$\begin{aligned} \dot{X} &= P_X + Y, \\ \dot{Y} &= P_Y - X, \\ \dot{P}_X &= P_Y - \frac{(1-\mu)(X-\mu)}{r_1^3} - \frac{\mu(X-\mu+1)}{r_2^3}, \\ \dot{P}_Y &= -P_X - \frac{(1-\mu)Y}{r_1^3} - \frac{\mu Y}{r_2^3}, \end{aligned} \tag{1}$$

   where

$$r_1^2 = (X-\mu)^2 + Y^2,$$
$$r_2^2 = (X-\mu+1)^2 + Y^2.$$

   Taking the Earth-Sun parameter $\mu = (3.040423398444176)\, 10^{-6}$

   (a) using standard numerical tools of your choice find three fixed points on the X axis,

   (b) using interval arithmetic and the interval Newton method verify that there truly exist fixed points inside given neighbourhoods of your numerical guesses.