rtg2010
August 12 - Challenges

1. Consider the Rössler equations

$$\begin{aligned}\dot{x} &= -(y+z)\\ \dot{y} &= x+by\\ \dot{z} &= b+z(x-a)\end{aligned}$$

with parameters $a = 5.7$ and $b = 0.2$.

   (a) On the section $V = \{x = 0, y < 0, \dot{x} > 0\}$ find sets N_1, N_2 which can be used as windows for a construction of a horseshoe of the map P^2 for the Poincaré map

$$P : V \to V.$$

   (The fact that P is strongly contracting in z coordinate to $z \approx 0.03$ will help in your construction)

   (b) Using interval arithmetic computations prove symbolic dynamics on N_1, N_2 for P^2.

2. Consider a damp forced pendulum equation

$$\ddot{x} + \beta\dot{x} + \sin(x) = \cos(t) \tag{1}$$

with $\beta = 0.2$. Equation (1) can be rewritten as an ODE

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= \cos(t) - \beta y - \sin(x)\\ \dot{t} &= 1.\end{aligned} \tag{2}$$

Consider a Poincaré section $V = \{t = 0\}$ of (2). Close to $(x, y) = (2.64, 0.063)$ there exists a fixed point (x_0, y_0) of the Poincaré map

$$P : V \to V.$$

(this fixed point is computed for you in the source file `pendulum.cpp` using the Newton method in function `orbit()`)

   (a) Sketch the unstable manifold of P. Find potential candidates for windows for a construction of a horseshoe for the map $f = P^k$ for some $k > 0$.

   (b) *Using interval arithmetic computations prove symbolic dynamics for f.