

Symmetries, patterns and orbifolds: activity sheet

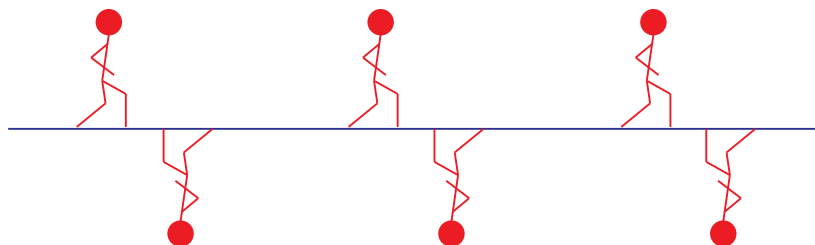
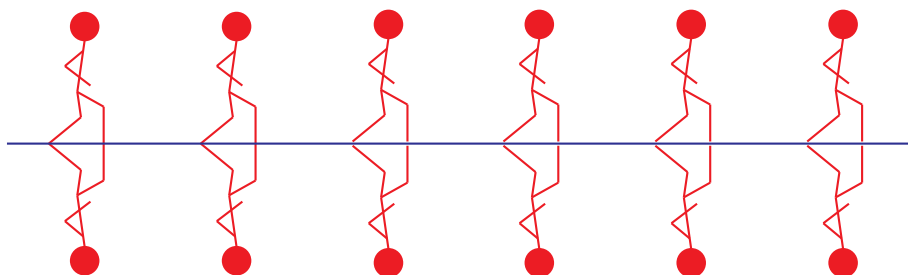
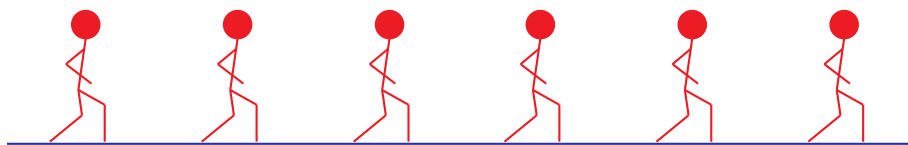
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1. Draw your own wallpaper patterns. Do you see any rotational symmetries? Reflections? Glides?
2. All four of these wallpaper patterns have a rectangular look to them, but only two of them have the same symmetries. Which ones?



3. Make your own orbifolds by drawing the frieze pattern on paper, then use paper scissors and tape to make the orbifold:



4. Here's a price-list for orbifold parts:

Feature	symbol	price (\$)
boundary	*	1
corner	$(*)n$	$(n - 1)/n$
cone-point	n	$(n - 1)/(2n)$
cylindrical end	∞	1
strip-like end	$(*)\infty$	1/2
handle	\circ	2
cross-cap	\times	1

Check that all these 17 signatures (which don't include the symbol ∞) cost exactly \$2:

$$*2222, *333, *442, *632, 2*22, 3*3, 4*2, 22*, \\ 2222, 333, 442, 632, **, * \times, \times \times, 22 \times, \circ$$

Do the same for these 7 signatures (which do include ∞ symbols)

$$\infty \infty, \infty *, * \infty \infty, 22 \infty, *22 \infty, \infty \times, * \infty \infty.$$

- What are the signatures for the 15 orbifolds depicted overleaf? (All of them are in the list of 17 from the previous question.) *Note:* the one that looks like a donut has signature \circ , but I didn't explain this symbol.
- A challenge: Suppose that we have some signature $*N_1N_2N_3 \dots N_k$ which costs exactly \$2. Show that k is either 3 or 4, and that if $k = 4$ the signature is $*2222$ while if $k = 3$, the signature is $*632$, $*442$ or $*333$. Can you prove that there are only 17 signatures (without ∞) that cost exactly \$2?
- What is the signature of the pattern made by the bathroom tiles in your home? What about the brickwork of the RLM building?

14 of the 17 wallpaper orbifolds

