EXERCISES

ARUN DEBRAY
JUNE 1, 2020

These problems aren't essential for this week's material, but could be helpful or at least interesting to think about. Do not feel the need to go in order or to do all of the problems, and pick the questions that look most interesting to you.

(1) We said during lecture that for $\mathcal{V}ect_{\mathbb{C}}^{\times}$, $\pi_0 = 0$, $\pi_1 = \mathbb{C}^\times$, and $k = 0$. Prove this.
(2) Likewise, we saw that for $s\mathcal{V}ect_{\mathbb{C}}^{\times}$, $\pi_0 = \mathbb{Z}/2$, $\pi_1 = \mathbb{C}^\times$, and k is the unique nonzero map $\mathbb{Z}/2 \to \mathbb{C}^\times$. Prove this.
(3) In lecture, we classified invertible field theories valued in $s\mathcal{V}ect_{\mathbb{C}}$. What if we used $\mathcal{V}ect_{\mathbb{C}}$ instead?
(4) Following the previous question: suppose that α is a bordism invariant defined by integrating a product of characteristic classes in ordinary cohomology (so a TFT of oriented or unoriented manifolds). Show that the invertible TFT it defines can be defined with target $\mathcal{V}ect_{\mathbb{C}}$ instead of $s\mathcal{V}ect_{\mathbb{C}}$.
(5) The *Arf invariant* $\Omega_2^{\mathrm{Spin}} \xrightarrow{\cong} \{\pm 1\} \hookrightarrow \mathbb{C}^\times$ is a complete bordism invariant of spin 2-manifolds. Therefore it categorifies to an invertible TFT $Z_A \colon \mathcal{B}ord_2^{\mathrm{Spin}} \to s\mathcal{V}ect_{\mathbb{C}}$. If you have seen the Arf invariant and/or are familiar with spin 1- and 2-manifolds: what are the state spaces of this TFT?