EXERCISES

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These problems aren't essential for this week's material, but could be helpful or at least interesting to think about. Do not feel the need to go in order or to do all of the problems, and pick the questions that look most interesting to you.

(1) Consider $\mathbb{Z}/n$-Dijkgraaf-Witten theory with cohomology class $\alpha = 0$. That is, perform the finite path integral summing principal $\mathbb{Z}/n$-bundles, beginning with the trivial theory for $Z^{c\ell}$. What is the partition function on a closed n-manifold? This theory is also called finite gauge theory or untwisted Dijkgraaf-Witten theory.

(2) $H^3(B\mathbb{Z}/2; \mathbb{R}/\mathbb{Z}) \cong \mathbb{Z}/2$, so there is a twisted Dijkgraaf-Witten theory with gauge group $\mathbb{Z}/2$ on 3-manifolds. The map $H^3(B\mathbb{Z}2; \{\pm 1\}) \to H^3(B\mathbb{Z}/2; \mathbb{R}/\mathbb{Z})$ is an isomorphism, so we could have done everything with $\{\pm 1\}$ coefficients, meaning this twisted Dijkgraaf-Witten theory extends to unoriented manifolds. Calculate the dimensions of the state spaces of this theory; you should obtain $\# H^1(\Sigma; \mathbb{Z}/2)$ if Σ is orientable and $\# H^1(\Sigma; \mathbb{Z}/2)/2$ if Σ is unorientable.

(3) Verify at the level of partition functions that bosonizing, then fermionizing, is equivalent to tensoring with an Euler theory.

(4) Think about the gauging and ungauging example (if it helps, restrict to dimension 2 for concreteness). What is the kernel invertible TFT? What happens if you gauge, then ungauge?