Solutions to Sample Midterm II

(1) $x^2 - 2xy + y^3 = 1$

(a) $P = (2,1)$

$2^2 - 2 \cdot 2 \cdot 1 + 1^3 = 4 - 4 + 1 = 1$

so P is a point on the curve.

(b) $2x - 2y - 2x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx}(3y^2 - 2x) = 2y - 2x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2y - 2x}{3y^2 - 2x}$

$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{P=(2,1)} = \frac{2 \cdot 1 - 2 \cdot 2}{3 \cdot 1 - 2 \cdot 2}$

$= \frac{2 - 4}{3 - 4} = 2$

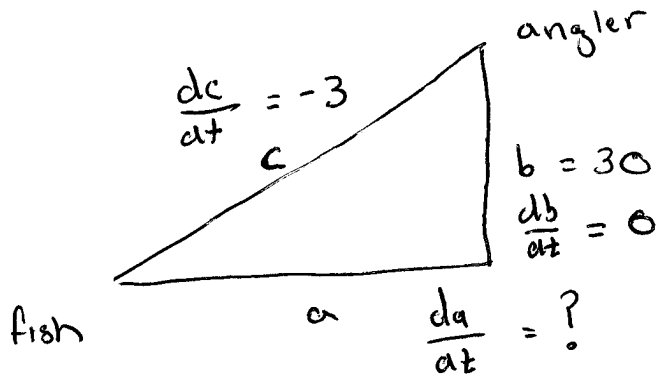
$\boxed{y - 1 = 2(x - 2)}$

Meets x-axis when $y = 0$

$$0 - 1 = 2(x - 2)$$

$$-\frac{1}{2} + 2 = \boxed{\frac{3}{2} = x}$$

(2)



$$c^2 = a^2 + b^2$$

$$\Rightarrow 2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$$ ($\frac{db}{dt} \to 0$)

when $c = 40$,

$$40^2 = a^2 + 30^2 \Rightarrow a = \sqrt{40^2 - 30^2}$$

$$2 \cdot 40 \cdot (-3) = 2\sqrt{40^2 - 30^2}\,\frac{da}{dt}$$

$$\Rightarrow \boxed{\frac{da}{dt} = \frac{-120}{\sqrt{40^2 - 30^2}} \text{ ft/s}}$$

In words: The length a between the fish and dock is decreasing at $\frac{120}{\sqrt{40^2 - 30^2}}$ ft/s

(3) $f(x) = x^3 + ax^2 + bx + c$

$0 = f(1) = 1 + a + b + c$

$f'(x) = 3x^2 + 2ax + b$

$0 = f'(1) = 3 + 2a + b \Rightarrow b = -3 - 2a \quad (\star)$

$0 = f'(-2) = 3 \cdot 4 + 2a(-2) + b$

$= 12 - 4a + b$

$= 12 - 4a - 3 - 2a \quad (\text{by } (\star))$

$= 9 - 6a$

$\Rightarrow \boxed{a = \frac{3}{2}}$

$b = -3 - 2\left(\frac{3}{2}\right) = \boxed{-6}$

$c = -1 - a - b = -1 - \frac{3}{2} + 6 = \boxed{\frac{7}{2}}$

(4) Optimization

Not on exam

(5) $f(x) = \dfrac{x-2}{x^2}$

(a) domain: $\mathbb{R} \setminus \{0\}$

vertical asymptote at $x = 0$

$\lim\limits_{x \to \pm\infty} \dfrac{x-2}{x^2} = 0$ since $2 = \deg(x^2) > \deg(x-2) = 1$

$\Rightarrow$ horizontal asymptotes at $y = 0$

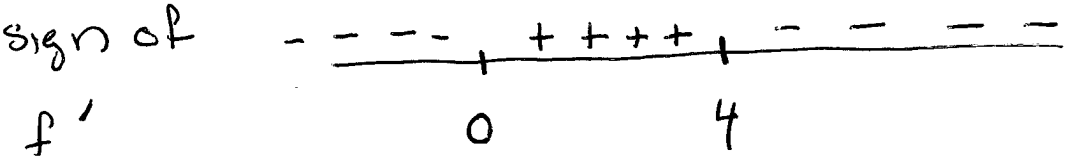
(b) $f'(x) = \dfrac{x^2 - 2(x-2)x}{x^4} = \dfrac{-x^2 + 4x}{x^4}$

$= \dfrac{-x+4}{x^3}$

$0 = f'(x) = \dfrac{-x+4}{x^3} \Rightarrow$ $\boxed{x = 4 \text{ is critical number}}$

Note: $x = 0$ is not critical number because 0 is not in domain

(c)



f decreasing on $(-\infty, 0) \cup (4, \infty)$

increasing on $(0, 4)$

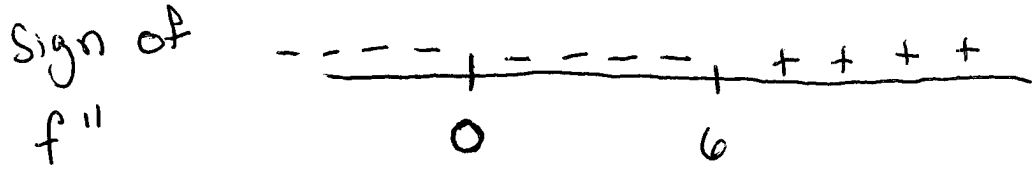
local max at $x = 4$

(6) (a) $$f''(x) = \frac{-x^3 - 3(-x+4)x^2}{x^6}$$

$$= \frac{2x^3 - 12x^2}{x^6}$$

$$= \frac{2x - 12}{x^4}$$

$$0 = f''(x) = \frac{2x-12}{x^4} \Rightarrow$$ **$x = 6$ is POI**



f concave down on $(-\infty, 0) \cup (0, 6)$

concave up on $(6, \infty)$

(b)

No y-intercept ($x = 0$)

x-intercept: $0 = f(x) = \dfrac{x-2}{x^2}$

$\Rightarrow \boxed{x = 2}$