373K Algebra I, Homework 10

From Artin

Chapter 7 (page 224) 7.2, 8.1, 8.3, 8.5.

Chapter 11 (page 354) 1.6, 1.7, 1.8.

Others:

1. Prove that there is no simple group of order 56.

2. Finish the proof that there are no non-abelian simple groups of order $n \leq 100$ with $n \neq 60$ (i.e. finish the product of three primes case).

3. Prove that if $H \neq 1$ is a normal subgroup of A_5 then H contains a 3-cycle.

4. Prove that if $|G| = 60$ and G contains more than one Sylow 5-subgroup then G is simple.

5. Prove that a group of order 108 has a normal subgroup of order 9 or 27.

6.(**Tricky**) Suppose that $|G| = 231 = 3.7.11$. Show that the Sylow 7 and 11 subgroups are normal and that the Sylows 11-subgroup is contained in $Z(G)$.

7. Let G be a finite group, H a normal subgroup of G and P a Sylow p-subgroup of H. Show that $G = HN_G(P)$ and $[G : H]$ divides $|N_G(P)|$. (This is known as *Frattini's argument.*)

8. Let $\mathbf{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbf{Z}\}$. Prove that this ring has infintely many units.

9. Let R be a commutative ring with 1, and let $R[x] = \{\text{polynomials with coefficients in } R\}$.

(a) Prove that $R[x]$ is a commutative ring with 1.

(b) Prove that if R is an integral domain, then $R[x]$ is an integral domain.

(c) What are the units in $R[x]$?