373K Algebra I, Homework 1

From Artin

Chapter 2 (pp. 69–73): 2.3, 2.6, 7.1, 7.6, 9.2, 9.4.

Others:

1. Suppose that $f : X \to Y$ has an inverse g, prove that f is a bijection.

2. Suppose that X and Y are finite sets with the same number of elements. Show that the following are equivalent:

(i) f is injective, (ii) f is bijective, (iii) f is surjective.

3. Let S be a set and S^* the set of all subsets of S. Define operations $+$ and $\cdot$ on S^* as follows: For $A, B, C \in S^*$ define:

$$A + B = (A \setminus B) \cup (B \setminus A) \text{ (this is the symmetric difference).}$$

$$A \cdot B = A \cap B$$

Prove that

(i) $(A + B) + C = A + (B + C)$.

(ii) $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$.

(iii) $A.A = A$

(iv) $A + A = \emptyset$

4. Give an example of a relation that is not reflexive (resp. symmetric, resp. transitive).

5. Let X be the set of polynomials with real coefficients. Define $\sim$ on X by $f(x) \sim g(x)$ if $f'(x) = g'(x)$.

Prove that $\sim$ is an equivalence relation.

Describe a complete set of equivalence classes.

6. If G is a group prove that the only element $g \in G$ with $g^2 = g$ is the identity element.

7. Let G be the set of rational numbers with odd denominators. For $a, b \in G$ define $a \cdot b = a + b$. Is $(G, \cdot)$ a group?