EE 381M, 16875
Probability & Stochastic Processes II
Coupling in Markov Chains, Random Graphs,
Branching and Queuing Processes
Fall 2018

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Objectives

The course is focused on the use of coupling techniques on a wide set of questions in stochastic processes. The main processes to be considered are Markov processes, branching processes, random graphs, and queuing processes. We will also discuss applications to network science and communications science.

Prerequisites

EE 381J Probability and Stochastic Processes I is a pre-requisite. The course will provide self contained material on Markov processes, random graphs, branching and queuing processes.

Contents

The course is structured in several blocks.

I. Markov Chains

This block will cover:

- Countable state space Markov chains;
- Total variation distance;
- Doeblin coupling;
- Coupling from the past;
- Coupling of renewal processes;
- Coupling in the continuum.

II. Random Graphs and Branching Processes

This block will discuss both random graphs in the Erdős–Renyi sense and infinite random sparse graphs in the Aldous and Lyons sense. The following notions will be covered:

- The Erdős–Renyi model;
- Branching processes;

- Coupling between random graph explorations and branching processes;
- Stein-Chen's method;
- Sparse infinite random graphs and networks;
- Mass transport, unimodularity;
- Percolation.

The following applications will be discussed: epidemic diffusion, viral processes, connectivity in communication networks, navigation on random networks, random walks on random graphs and trees.

III. Queuing Dynamics

Coupling techniques for stochastic recurrences. The following topics will be discussed

- Birkhoff's point wise ergodic theorem;
- Loynes' coupling construction for single server queues;
- Loynes' coupling construction for multi-server queues;
- Loynes' coupling construction for queuing networks;
- Point processes on the real line;
- Palm calculus;
- Coupling representation of Palm probability;
- Rate conservation principle.

Applications will include protocol analysis and advanced queuing network analysis: conservation rules, priority queuing, spatial queuing.

References

The following material will be used in the course:

[BB] F. Baccelli & P. Brémaud, *Elements of Queueing Theory*, Springer Verlag, second edition, 2003.

[Bol] B. Bollobas, *Random Graphs*, Cambridge University Press, 2001.

[DM] M. Draief and L. Massoulié, *Epidemics and Rumors in Complex Networks*, London Mathematical Society Lecture Note Series, 2010.

[Gri] G. Grimmett, *Random Processes on Graphs and Lattices*, Cambridge University Press, 2010.

[LP] R. Lyons and Y. Peres, *Probability on Trees and Networks.* On line.

[VdH] Remco van der Hofstad, *Random Graphs and Complex Networks,* Cambridge, 2016.

[Tho] H. Thorisson, *Coupling, Stationarity, and Regeneration*, Springer, 2000.

Grading

- Assignments: 1/3;
- 2 midterm exams: 1/3;
- One research paper to read and present (from a list of proposed papers): 1/3.