$R = \mathbb{C}[x_1, \ldots, x_n]$ or $\mathbb{C}[x_n, \ldots, x_1]$

$R_f = R/(\partial f)$ finite dimensional (ie regular sequence $\frac{\partial f}{\partial x_i}$)

$\text{MF}_f$ category of matrix factorization

with $\text{Hom}_{\text{MF}}(M,N)$ is an $R$-module

$\text{HMF}$ - homotopy category of matrix factorizations

triangulated category

$\text{Hom}_{\text{HMF}}(M,N)$ is an $R$-module, dually

since $D^2 = \mathbf{1}$ $\Rightarrow \frac{\partial D}{\partial x} \frac{\partial D}{\partial x} = 2f \mathbf{1}$

so $\frac{\partial f}{\partial x}$ is homotopic to the identity

Theorem (R.O. Buchweitz) $\text{HMF}$ is a

Cohomology category:

$\text{Hom}_{\text{HMF}}(M,N) \times \text{Hom}(N,M) \to \mathbb{C}$

natural cochain pairing: $m$ is even
If an odd natural pairing \( (\alpha, \beta) \mapsto [H(\alpha \beta)]_{\mathfrak{m}} \) is 1.

Also \( K(\mathcal{H}M) \) is (believe to be) torsion? ??

Can read off a factorization by the maximal ideal:

\[
\begin{align*}
M^0/\mathfrak{m}^0 & \xrightarrow{D} M^1/\mathfrak{m}^1 \xrightarrow{D} M^2/\mathfrak{m}^2 \\
H(M) & = \text{homology of this complex.}
\end{align*}
\]

A map \( \alpha : M \rightarrow N \) is an isomorphism if

\( H(\alpha) : H(M) \rightarrow H(N) \) is an isomorphism.

… so analogy of homology & of derived category.

Category is Katz-Schmid: every object is \( \mathbb{Q} \)-acyclic.

A unique way to represent.

Let \( \mathbb{R}_{x,y} \) be the factorization

\[
\begin{align*}
R & \xrightarrow{x+y} R \\
\otimes (x) & = x \cdot y
\end{align*}
\]

Then \( M \otimes \mathbb{R}_{x,y} = M_x \) locally finitely

\[
\begin{align*}
\mathbb{R} & \xrightarrow{y} \mathbb{R} \\
\otimes (x) & = x \cdot y
\end{align*}
\]
The picture for any factorization of
\[ f = \sum f_i \]
can be to \( \sum \) over common polynomial.

\[ \implies M \otimes N \text{ matrix factorizations} \]
\[ \sum_{x_i, y_i} \text{ of } (x_i, y_i) \text{ sum polynomial.} \]

Or \( \implies M \otimes N \rightarrow \odot M(x, y) \)

More generally, if we have \( f(x, \ldots, y) \),
\[ f(x, \ldots, y) - f(x, \ldots, 1) = \sum (x_i, y_i) \text{!} \]
\[ \implies L_{xy} = \odot (R \xrightarrow{xy} R) \text{!} \]

represents the identity (analogy of diagonal):
\[ M @ L_{xy} = M \]
To $f = x^{n-1}$ assign extended 2d TQFT

\[ \Delta \rightarrow \mathbb{C}^n \times \mathbb{C}^n \]

\[ \Theta \rightarrow \text{Rep} \cong \mathbb{C}[x]/x^n \]

\[ \begin{array}{c}
\tau \rightarrow \tau^2 \\
\psi \rightarrow \psi^3
\end{array} \]

\[ x \delta_x R \xrightarrow{\sim} R \xrightarrow{\sim} R \]

\[ R \xrightarrow{\sim} \mathbb{C} \xrightarrow{\sim} \mathbb{C} \]

\[ \text{List by: we assigned } \chi^2 = (\chi \gamma(\sigma^{(x)}) \gamma(x)) \]

\[ \chi = (\chi \gamma(\sigma^{(x)}) \gamma(x)) \]

To crossing the assign complexes of formal:

\[ 0 \rightarrow X \rightarrow \mathcal{T} \rightarrow 0 \]

\[ 0 \rightarrow \mathcal{T} \rightarrow 0 \]

\[ 0 \rightarrow (\mathcal{T}) \rightarrow \mathcal{T} \rightarrow 0 \]
For maps of complexes, consider homology in the category of HMFs.

Find in here Redei nice ones not satisfied.

\( \varphi \sim \psi \) do

\( F(\chi) \) to tasks with orientation.

[Need check if not: want x not to have nonzero degree ... ?]

\( \Rightarrow \) big idea: homology theory for links

\[ K^2 P(U) \xrightarrow{\varphi^*} P(U) \xrightarrow{\partial} (2, \varphi^*) \]

\( \varphi \) homological potential of point \( \chi \)

\( o \sim g_l \sim g_l \sim 0 \) which vanishes satisfying Redei nice.
To any graph \( \Gamma \) with given boundary

one can assign an invariant tensor

\[ m(\Gamma) \in \text{Inv}(V_0 V_1 V_2 \ldots V_r) \]

After categorification get \( M(\Gamma) \in \text{HMF} \)

but this has a bad K-groups ...

but can still obtain category by summing over all such graphs,

\[ A : \text{\#} \text{Hom}(M(\Gamma), M(\hat{\Gamma})) \]

with I some (conjecturally finite) collection of \( \Gamma \) with \( m(\Gamma) \) spanning space of invariants

\[ \Rightarrow \text{conjecturally f.g. proj A-modules } \subset \text{HMF} \]

Tangle: start categorify space of invariants of linear pencils

\[ \text{K(C)} \text{-- ideals} \]

\[ \text{(C)} \]
Conjecture: Direct sums of $M^2$ ($\text{Vor}$ $\text{P}$)

Some basis dual canonical basis in kernel $\text{End}$ $(V_0 \ldots V_0 \ldots)$

Replace $C(\text{HRF})$ with $C^\infty$, not $C_0$.

Equation $\text{Am} = 0$ may not include things in $R^2$. 

But $\text{Am}$ may not include $R^2$. 