M. Douglas - Microscopic theory of D-branes

Note Title 11/30/2007

(X, g) Riemannian manifold
$M \subset X$ submanifold with vector bundle E
& connection ∇, impose extra conditions $\rightsquigarrow$ get D-branes.

e.g. X Kähler, M, E holomorphic
∇ hermitian Yang-Mills $\begin{cases} F^{0,2} = 0 \\ \omega^{d-1} \wedge F^{1,1} = c\,\omega^d \end{cases}$

Donaldson-Uhlenbeck-Yau: solution ∇ exists iff E is μ-stable (subsheaves are decreasing in slope) Necessity easy, sufficiency need to get into microscopic geometry (PDE)

In large volume these will exactly be the branes: coherent sheaves with hermitian YM connections. But in small volume regime more complicated — conjecturally objects in the derived category! [Kontsevich]

Why derived category? GSO projection in $N=2$ SCFT: Fermion number is not just in $\mathbb{Z}/2$ but have an integer $U(1)$ charge.

Quasiisomorphisms: respect correlation functions of vertex operators (traces of products of morphisms) – so won't detect anything finer than quasiisomorphism physically!

What is HYM equation for $E \in D(\mathrm{Coh}\,X)$? What about Donaldson-Uhlenbeck-Yau theorem?

Conjecture (Π-stability, Bridgeland stability): notion of stability (slope) so that in triangles to have all three objects stable need $\varphi(E') < \varphi(E)$.

$$\begin{tikzcd} E' \arrow[r] & E \arrow[dl] \\ E'' \arrow[u, "{[1]}"] & \end{tikzcd}$$

Don't know very well what the $\underline{\text{equation}}$ (HYM type) should be $\rightsquigarrow$ need microscopic definition [ie know necessity not sufficiency]

Microscopic definitions:

1. $\underline{\text{boundary states}}$ in 2d CFT
2. σ-model and renormalization group
3. String field theory

(0. Equations on complexes...)

Consider special case $X = T^{2d} = \mathbb{C}^d/\Lambda$
+ B field. Can write down solution of string theory explicitly & rigorously — can we make sense of D-branes in this?

$X = T^{2d}$ $\quad$ $\mathcal{H}$ = Hilbert space assoc. to S^1
(closed string states)

$$\mathcal{H} = \bigoplus_i \mathcal{H}_i^{(L)} \otimes \mathcal{H}_i^{(R)} \quad \text{reps of } Vir_L \otimes Vir_R$$

controlled sum.

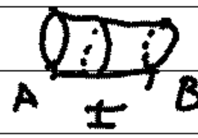
Here we embed $Vir_L \otimes Vir_R$ in Heisenberg algebra $U(1)_L^{2d} \otimes U(1)_R^{2n}$: Fourier-type decomposition of strings on T^{2d} (winding & momenta)

D-branes: associated to boundary conditions on intervals $_A[\!\longrightarrow_B$: want to define states on such intervals.

Consistency conditions:

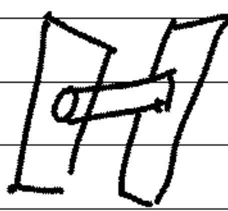
• Virasoro actions must match up on the boundary. Consider $I \times S^1$  :

Can consider as a trace on the open string Hilbert space

time ↑ 

$\mathrm{Tr}_{\mathrm{Hom}}\, e^{-tH}$ (*)

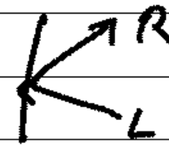
OR as time evolution on the circle



Create D-brane, propagate for some time & kill it

ie boundary state $\langle A | e^{-\frac{1}{t} H_{cl}} | B \rangle$

$$|B\rangle \in \underset{\mathrm{Hom}(\mathcal{H}^L, \mathcal{H}^R)}{\overset{\shortparallel}{L}} \supset \mathcal{H}_{S^1}$$



more for nonzero time get honest state in $\mathcal{H}_{S^1}$

Virasoro consistency:

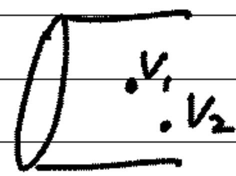
$$0 = (L_n^L - L_{-n}^R)|B\rangle$$

(*) Equality above is the Cardy condition...

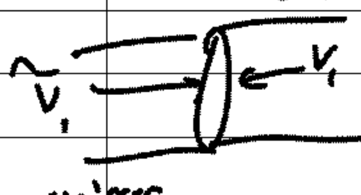
Can impose also invariance under current group, or other kinds of constraints.

• Classifying algebra (Cardy, Lewellen, Friedan, Gaberdiel, ..)

Given boundary



Construct commutative associative algebra with basis the Hilbert space: take limits



singularities of bulk operators multiply boundary operators — but we'll just take the leading term corresponding to the unit operator:

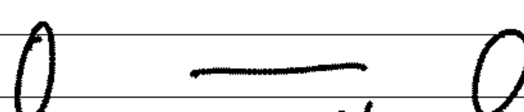
boundary state := module for this bulk algebra, commutative associative action

Space of D-branes = spectrum of this classifying algebra ... explicitly written in terms of braiding & fusing for $c<1$.

$c<1$ this works

$c=1$: generic case S^1 with irrational radius

spectrum is two S^1's & an interval



D0s: points on S^1 "special states" D1s: flat connections on S^1

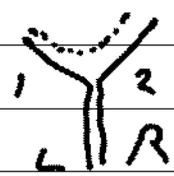
Special states fail Cardy condition!
So at $c=1$ expect space of D-branes
is only a <u>subset</u> of Spec C

SU_2 level 1: D0 branes $\longleftrightarrow$ points of SU_2
but Spec $C = SL_2\mathbb{C}$

$c>1$??

3. String field theory.
Introduce string field $A \in \mathbb{C}[\mathrm{Map}(I, X)]$
with given boundary conditions

Multiplication law: convolution over midpoint of strings
version of OPE

$A_1 * A_2 =$ 

$\Rightarrow$ get flatness condition
$$QA + A * A = 0$$
Q: BRST operator

Martin Schnabl - find some exact solutions!