Northwestern Low Spaces

D. Nadel. Proved: relative local geodesic

Local geodesic longods concern geodesic $L$

Relation of LG to G: $S'$ fixed points.

$\mathbb{M}$ compact space with $S'$ action

Morita $H^\ast_S(M) \cong H^\ast(M^{S'})$:

$E/\sim = BS' = CP^\infty$

$E$ contractible space with free $S'$ action
Categorical version:

\[ D^b_c(M) \otimes C\mathcal{U}^c = D^b_c(M^\heartsuit) \otimes C\mathcal{U}^c \]

(follows from GKM)

On the Lagrangian side, we're interested in \( h_c \)-equivariant cohomology on Springer & Steinberg varieties:

\[ G = \{ (g, B) \in G^* \times B^* : g \in B \} \]

\[ S^1 = G \times \mathbb{C}^* = \{ (g, B, B_1) : g \in B, B_1 \} \]

Doesn't look like a braid space...

What could be the analog of Segaloid localisation in loc?

Derived loop spaces

Would like to make sense of \( \mathcal{L}X = \text{Map}(S^1, X) \)

in world of algebraic geometry — \( X \) a scheme or stack.

First approximation: \( X \) a stack, e.g. \( X = \mathbb{Z}/\Gamma \)

Let \( g \rightarrow \Lambda \) : space where parts have automorphisms
\( S^1 \to X \): too quick to correct different parts

of \( X \) (not homotopy invariant)

but can zip around on automorphism

\[ \Rightarrow \quad IX = \left\{ \left( x, g \right) \mid x \in X \text{ and } g \in \text{aut } X \right\} \]

\( \text{inertia stack} \)

One restriction:

\[ \text{rings } \rightarrow \text{ spectra} \]

really just see fibrations spaces: \( \text{II } K(\mathbb{F}, 1) \)'s

\( S^1 \) also defines such a factor, look

at homotopy classes of maps \( \to IX \)

\[ \text{e.g. } X = \mathbb{C} \quad IX = G/G \text{ cofiber yuck} \]

Another attempt: \( S^1 \to \ast \)

Map \( S^1 \to X \) a smooth slice:

two points in \( X \), & they're equal, & key is equal.

\[ \Rightarrow \quad IX = X \times X \]

\[ X \times X \]
How to interpret well this Nakamura intersection? $\mathcal{O}_X \cong \mathcal{O}_x \oplus \mathcal{O}_x$

should be derived: $\mathcal{O}_{2x} \cong \mathcal{O}_x \oplus \mathcal{O}_x$

= Hochschill homology sheaf.

Calculate by Koszul complex $\Rightarrow$

$\mathrm{H}^1 \cong \mathcal{O}_x \otimes \Omega^1\big|_X$

$L_X = T_X[-1] \text{ odd tangent bundle}$

usual answer from physics or supergravity

$\text{Replace } S^1 \text{ by } S^3,$

$H^2(S^3) = \mathbb{C}[\eta]/\eta^3 = \mathbb{C}[\eta^3] \quad \eta^3 = 0: \text{ odd algebra}$

... works ?? in supergravity... odd lie $M^{11}$

Map to $M^{11}, X = T_X[-1] \text{ odd tangent bundle}$

Vee does the answer lie? DAG

\[
\begin{array}{ccc}
S^1 & \to & S^3 \\
\downarrow & & \\
S^1 & \to & S^3
\end{array}
\]
For general (smooth) stacks \( \mathcal{X} \)

is combinator of odd facets & inertia.

\[
\tilde{\pi}_k[-] : \mathcal{X} \to \mathcal{X}
\]

small base large

\( \mathcal{G}/\mathcal{G} \to \mathcal{G}/\mathcal{G} \)

\( S' \to \mathcal{X} \). On \( \tilde{\pi}_k[-] : \) the derivative

of this action is the action of \( H_k(S') \)

an odd vector field, the deRham differential

\( \to \) cyclic stratification of \( HH \) gives

Carron differential

\[ \text{Categorify:} \quad HH \to \text{Coh} \mathcal{X} \]

\[ HH^P \to \text{Coh}([X]^{\text{st}}, \bullet) \]

\[ \text{Theorem:} \quad D(H(X, D)^{\text{st}} \otimes \mathcal{C}_X^{\text{per}}, \bullet^2) \]

\[ = D(X, D) \otimes \mathcal{C}_X^{\text{per}} \]

(\text{unlaoced version } \to \text{RED-modules})
Applicability: \( X = \frac{G/G^\nu}{G^0/G^\nu} \)

\( \delta X = \frac{\delta^\nu}{\delta^0} \rightarrow H^0 \rightarrow H^\nu \)

Corollary: \( D \left( \frac{G^\nu}{G^0} \right) \circ \sigma \left( \gamma \right) \)

\( = D\left( \frac{\delta^\nu}{\delta^0} \right) \circ \sigma \left( \gamma \right) \)

ie (the) longest product for LG &

" " " " \( \mathbf{G} \) related

by sue process - \( \gamma \) (coaction) -

as \( \mathbf{G} \) representations.