RESEARCH STATEMENT

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1. Overview

I am interested in the interface between **representation theory**, **algebraic geometry** and **mathematical physics**. The main focus of my current research is a new approach to the representation theory of real and complex Lie groups built from a combination of the **geometric Langlands program**, **topological field theory** and **derived algebraic geometry** [BNa1, BFN, BNa2, BNa3, BNa4]. Another program I am working on applies **noncommutative geometry** [BNe2, BNe5, BNe6], in particular the geometry of differential operators [BNe1, BNe4], to questions concerning **integrable systems** [BF1, BNe3]. I am also interested in the applications of algebraic structures coming from physics (such as conformal field theory and **vertex algebras** [FB, BF2, BHS]) to the geometry of **moduli spaces** [BB1, BB2].

The geometric Langlands program is a nonabelian Fourier theory for the representations of loop groups, and an aspect of the electric-magnetic duality of 4-dimensional gauge theories. In my papers with D. Nadler, we find that this fundamental duality has far-reaching consequences for classical questions in representation theory. Mathematically, the new underlying principle is that the representation theory of Lie groups is recovered from the representation theory of the corresponding loop groups. The mechanism for this recovery is a sophisticated homotopical form of taking invariants for loop rotation [BNa1], formulated using the emerging field of derived algebraic geometry [L]. Physically, the dimensional reduction of electric-magnetic duality gives rise to a previously unexplored duality for 3-dimensional Chern-Simons type topological field theories. Using a homotopical theory of centers of tensor derived categories (which we develop with J. Francis in [BFN]), we find in [BNa2] a new topological field theory controlling the theory of character sheaves (Lusztig's geometrization of characters) for complex groups and an unexpected equivalence of these character theories for Langlands dual groups. This theory has tantalizing connections with Khovanov's link homology and the theory of Macdonald polynomials. For real groups we find a conceptual approach to the mysterious combinatorics of Harish-Chandra representations [BNa4], including Vogan's character duality and its generalizations [BNa3], and prospects for a complete understanding of the real component of the classical Langlands program (see [S2]).

The theory of integrable systems is a close relative of representation theory with a unique appeal and ubiquitous surprise appearances. Much of my work has been concerned with finding and exploiting geometric explanations for phenomena of integrability, as epitomized by the role of the Korteweg-de Vries (KdV) equation in the

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theory of algebraic curves (which I generalized in my thesis work with E. Frenkel [BF1]). My papers with T. Nevins have developed and explored relations between integrable systems and the theory of modules over rings of differential operators ($\mathcal{D}$-modules). In [BNe2] we showed that the noncommutative geometry of $\mathcal{D}$-modules controls the poles of solutions to soliton equations (such as the KdV and KP hierarchies). This connection enabled us to explain a well known and long-standing puzzle of integrable systems: poles of meromorphic solutions to soliton equations obey simple particle systems. In ongoing work [BNe5] we extend this program from $\mathcal{D}$-modules to modules over rings of difference operators, greatly extending its range of applicability (in particular to behavior of solitons of Toda lattice hierarchies). As part of this general project, we showed in [BNe1] that $\mathcal{D}$-modules were unexpectedly well behaved on the class of varieties with cusp singularities, while in [BNe4] we used noncommutative algebraic geometry to give a complete and explicit description of the moduli spaces of $\mathcal{D}$-modules on curves. We have used the techniques we developed these papers to formulate a broad program to geometrize elliptic representation theory. This is the study (pioneered by Cherednik, Felder, Etingof and others) of the fascinating algebras and integrable systems attached to moduli of bundles on elliptic curves. Most recently, the theory of integrable systems built from $\mathcal{D}$-modules which we have pioneered has been connected by R. Dijkgraaf and collaborators [D] to the Dijkgraaf-Vafa theory giving integrable structures in topological string theory [ADKMV], suggesting exciting new questions and applications for our future work.

Below I describe in more detail the two main directions of my current research. Please see the annotated bibliography for abstracts of all my papers.

2. Representation Theory and Topological Field Theory

In this section I outline my research program with D. Nadler aimed at advancing our understanding of the representations of Lie groups by combining results from the geometric Langlands program, techniques from derived algebraic geometry and structures from topological field theory.

2.1. Loop Groups and Complex Groups.

Modern geometric representation theory, in the hands of Beilinson, Drinfeld, Lusztig, and others, has seen the emergence of a categorified geometric analogue of harmonic analysis. In this setting categories of sheaves take the place of the function spaces of classical harmonic analysis, Fourier-Mukai transforms replace the Fourier transform, character sheaves replace characters, Hecke functors replace Hecke operators and so on. The geometric Langlands program describes an analogue in this sense of the Langlands program in which groups $G(K)$ over local fields K are replaced by loop groups $LG = Map(S^1, G)$ and locally symmetric spaces are replaced by moduli spaces of G-bundles on Riemann surfaces. In its local form, it proposes a kind of nonabelian analog of the Fourier transform or Pontrjagin duality, parametrizing the representation theory of the loop group LG in terms of the geometry of the *Langlands dual group* $G^\vee$ (a complex Lie group with dual maximal torus to G), much as the Fourier transform decomposes functions on a topological group in terms of its group of characters. (See the book by Frenkel [F] for an overview of the geometric Langlands program with an emphasis on the local version of Frenkel-Gaitsgory [FG] for loop groups.)

Our work has uncovered applications of the geometric Langlands program to the representation theory of real and complex Lie groups, in other words to the archimedean part of the classical local Langlands program. A natural symmetry of loop groups is the action of the circle rotating loops, whose fixed points are the corresponding finite dimensional groups. We show in [BNa1, BNa2] how this symmetry interacts with the geometric Langlands duality, specifically with one of its strongest local forms, due to Bezrukavnikov [Bez]. We develop in [BNa1] a sophisticated form of the principle relating localized equivariant cohomology to the cohomology of fixed points, using techniques borrowed from homotopy theory via the emerging discipline of derived algebraic geometry[1] [L]. In particular we develop a new description of categories of modules over differential operators ($\mathcal{D}$-modules) as equivariant localizations of categories of coherent sheaves on loop spaces.

The result of these developments is an unexpected form of Fourier duality for complex Lie groups. First we prove the strongest known form of the complex local Langlands program:

Theorem 2.1. [BNa1, BNa2] There is a canonical monoidal equivalence[2] of the derived categories of infinite-dimensional (Harish-Chandra) representations for the complex Lie group G and its Langlands dual $G^\vee$.

This result improves on the strongest results on representations of complex groups, due to Beilinson-Ginzburg-Soergel [BGS, S1] in that it preserves the structure of convolution, or action of intertwining operators, on the two sides (which are manifestations of the braid group, see below). This improvement allows us to show in [BNa2] that the theories of character sheaves, Lusztig's categorified form of characters, for G and $G^\vee$ are equivalent:

Theorem 2.2. [BNa2] There is a canonical equivalence of braided (or ribbon) tensor categories[3] between the derived categories of character sheaves for a complex group G and its Langlands dual group $G^\vee$.

To establish this result we first develop in [BFN] (with J. Francis) a notion of categorical center, or Drinfeld double, in a general homotopical setting. We then calculate the centers of the categories of representations of G and $G^\vee$ appearing in the previous theorem and show they give the (2-periodic) categories of character sheaves.

2.2. **Real Groups and Base Change.** A driving motivation behind our work is its applicability to the intricacies of the representation theory of real Lie groups.

The classical Langlands program gives a strong organizing principle for the representations of real groups, the archimedean local Langlands program. In the '70s Langlands

[1]As usual, equivariant localization produces 2-periodic cohomology groups, a point we suppress in this informal overview.

[2]More precisely, there is an equivalence between representations with generalized infinitesimal characters for G and their strict equivariant counterparts for $G^\vee$, the finite Hecke categories.

[3]More precisely, this is an equivalence preserving the homotopical versions of braided structure, known as an E_2-structure, and of ribbon structure.

gave a parametrization of the irreducible representations of real semisimple or reductive groups $G_{\mathbf{R}}$ in terms of complex geometry of the dual group $G^\vee$, reformulating work of Harish-Chandra from the '60s. In the '80s a striking result of Vogan [V] (developed further in [ABV]) uses an explicit analysis of the combinatorics of real groups to enhance this classification to a Langlands dual description of the K-group of representations of $G_{\mathbf{R}}$. Most recently, in the '90s Soergel [S1, S2] conjectured that Vogan duality is a reflection of a deeper underlying equivalence of derived categories of (Harish-Chandra) representations, using his result mentioned above in the complex case as the strongest piece of evidence.

In [BNa4], we show that the geometric Langlands philosophy provides a conceptual approach to the complicated structure of real group representations. First we describe a geometric form of Langlands' base change conjecture, which relates the geometric Langlands categories on a Riemann surface and its covering spaces. Geometric base change allows one to extend geometric Langlands to unoriented surfaces such as $\mathbb{RP}^2$ (considered as the quotient of $\mathbb{CP}^1$ by the antipodal map). In that case we show that the S^1 equivariant localization described above precisely produces (a strengthened form of) Soergel's local Langlands conjectures from geometric Langlands on $\mathbb{CP}^1$:

Theorem 2.3. [BNa4] The geometric base change conjecture on $\mathbb{CP}^1$ implies Soergel's conjecture.

In other words, the intricate combinatorics of real group representation theory is conjecturally explained geometrically by the combination of base change and equivariant localization. As a demonstration and application of these ideas we are able to prove our conjectures on the level of K-theory in [BNa3]. As a result we obtain a purely geometric proof of Vogan's character duality, and in fact deduce it from a new result which can be considered as a form of Vogan duality for loop groups, or as a real form of Kazhdan-Lusztig's construction of affine Hecke algebras:

Theorem 2.4. [BNa3] Geometric base change holds on the level of K-groups for the antipodal map on $\mathbb{CP}^1$, giving a canonical duality (as modules for the affine Hecke algebra) between the K-groups of constructible sheaves on the moduli of real G-bundles and of coherent sheaves on the space of Langlands parameters. Moreover Vogan's character duality is obtained as the S^1-equivariant localization of this duality.

Our continuing work is aimed at proving the geometric base change conjecture, hence in particular Soergel's conjecture, in general.

2.3. **Topological Field Theory.** Much of my thinking about geometric representation theory is guided by structures arising from topological and conformal field theory (the latter of which was the subject of some of my earlier work, [FB, BF2, BHS]). The most exciting development in this direction is the discovery by Kapustin and Witten [KW] that the geometric Langlands program appears naturally as a two-dimensional aspect of the electric-magnetic duality of four-dimensional gauge theories. Namely Kapustin and Witten constructed (from a physical standpoint) a "magnetic" 4-dimensional topological field theory associated to the group G and and an "electric" theory associated to $G^\vee$. They then showed how the physics of branes and line operators in the theories (when compactified to two dimensions) recovers the categories of sheaves and Hecke operators of geometric Langlands, and showed that the equivalence of these two field theories (the Montonen-Olive S-duality conjecture) gives rise to the geometric Langlands duality. (My book in progress [BZ], based on lectures in Oxford and a course in Austin, provides in particular the first mathematical exposition of this work.)

The S^1 localization by which we relate representations of loop groups to representations of Lie groups arises naturally out of topological field theory considerations. Namely, we are studying the dimensional reductions of the electric and magnetic theories of [KW] from four to three dimensions. Our results in [BNa1] suggest that both theories dimensionally reduce to three-dimensional topological gauge theories (associated to G and $G^\vee$), which are related to the Chern-Simons theory for *complex* gauge groups, instead of the usual compact groups. We thus obtain a new physical prediction of an equivalence between complex Chern-Simons type theories for Langlands dual groups. More generally, our constructions in [BFN] give rise to a partial three-dimensional topological field theory associated to any derived stack (with the above related to the case of the classifying space BG). This theory is a categorified form of string topology, replacing cochains on loop spaces with derived categories of sheaves. I have been discussing these results with E. Witten, who has proposed that they be interpreted using a partial three-dimensional extension of supersymmetric sigma models.

The mathematically accessible part of the 3-dimensional reduction of the electric and magnetic theories is a smaller 3-dimensional field theory built out of character sheaves[4], which we call the character theory. Roughly speaking, our Langlands duality for character sheaves is an isomorphism of the character theories for G and $G^\vee$, a new 3-dimensional manifestation of electric-magnetic duality. A "Hodge" version of the character theory also promises to lead to a proof of some conjectures of [H] on Macdonald polynomials and the cohomology of representation (or character) varieties for Riemann surface groups (which appears when applying the character theory to a closed surface).

Finally, another aspect of the character theory is a relation to Khovanov's link homology theory [Kh], which is also motivated by aspects of 4-dimensional gauge theory. Namely the category of Harish-Chandra modules for a complex Lie group G is a categorical form of the braid group of G, which has been intensively studied by Soergel, Rouquier and others. Khovanov showed that the study of Hochschild homology of objects in this category gives rise to link homology groups, of great current interest in low dimensional topology. Our results give a precise (definition of and) description of the center of this category, which gives a refined understanding of the Hochschild homology considered by Khovanov. Moreover our identification of these "braid categories" for G and $G^\vee$ suggests a Langlands duality for Khovanov homology, which we hope to explore.

3. Integrable Systems and Noncommutative Geometry

In this section I describe some themes developed in my recent work with T. Nevins on integrable systems and our program to geometrize elliptic representation theory.

[4]More precisely, this structure gives the 0,1 and 2 dimensional part of a 3-dimensional field theory, in particular (the homotopical form of) a ribbon tensor category.

3.1. **Solitons and Many-Body Systems.** Soliton equations are a remarkable class of completely integrable PDEs, including the KdV, KP and Toda lattice hierarchies, with numerous interconnections to algebraic geometry, representation theory, matrix models and string theory. A recurring theme in my work has been the search for conceptual geometric explanations of the diverse manifestations of solitons, starting with my thesis [BF1] with E. Frenkel on the geometry of Drinfeld and Sokolov's generalizations of KdV.

One of the more studied puzzles of the theory of integrable systems regards the phenomenon that the motion of the poles of meromorphic solutions to soliton equations tends to be governed by classical many-body systems [Kr1, Kr2, W]. In particular the KP equations (generalizing KdV) have rational, trigonometric and elliptic solutions, whose poles evolve under the flows of the KP hierarchy according to the corresponding CM systems. In [BNe2], Nevins and I give a thorough analysis and solution of the KP/CM correspondence puzzle and the first conceptual explanation for the phenomenon. In fact we prove a much stronger result:

Theorem 3.1. [BNe2] The Fourier-Mukai transform induces is a canonical isomorphism between the phase spaces of the rational, trigonometric and elliptic multicomponent KP hierarchies and of the corresponding spin Calogero–Moser particle system (completed so as to allow collisions of particles), identifying all flows and sending singularities of the KP Lax operators to positions of particles.

Our approach involves extending and geometrizing Sato's $\mathcal{D}$–module approach to KP by recasting it in the language of noncommutative geometry. This allows us to describe poles of solutions geometrically and to construct phase spaces as moduli spaces of torsion–free $\mathcal{D}$–modules on elliptic curves. To describe these moduli spaces we develop a generalization of the Fourier–Mukai transform. The resulting transform identifies (the derived category of) coherent $\mathcal{D}$–modules on an elliptic curve with (the derived category of) coherent sheaves on ruled surfaces. On the other hand in [BNe3] we show that coherent sheaves on these ruled surfaces naturally describe spin CM particle systems, completing the correspondence.

This work has had numerous outgrowths, including the investigation [BNe1] into general (and surprising) properties of $\mathcal{D}$-modules on a class of singular spaces (see the annotated bibliography). We have also been extending our techniques and results from differential operators to difference operators. The geometric theory of rings of difference operators and their modules is far less developed than its differential counterpart, giving rise to interesting geometric challenges we have been pursuing. In [BNe5] we develop a Fourier-Mukai transform for finite difference, q-difference and elliptic difference operators and use it to give a geometric particle system description of (nonabelian generalizations of the) Toda lattice hierarchies, extending the KP/CM correspondence of Theorem 3.1 while explaining and generalizing the Toda–Ruijsenaars correspondence of [KrZ].

We have also developed an explicit general description of $\mathcal{D}$-modules on curves in [BNe4], motivated by the theory of Cherednik algebras and answering a question of Ginzburg:

Theorem 3.2. [BNe4] All moduli spaces of $\mathcal{D}$-modules on a smooth curve X are given by twisted cotangent bundles to moduli stacks of (complexes of) coherent sheaves on

X (with twist given by the determinant line bundle). The corresponding (untwisted) cotangent spaces are the moduli of perverse coherent sheaves on T^*X.

In the case of torsion-free $\mathcal{D}$-modules ("$\mathcal{D}$-bundles") this gives a quiver-type description of their moduli, generalizing and explaining the identification of the space of ideals in the Weyl algebra with Calogero-Moser quiver varieties due to several authors. The corresponding moduli of "perverse vector bundles" on T^*X contain as open subsets the moduli of framed torsion-free sheaves (e.g. the Hilbert schemes $(T^*X)^{[n]}$ in the rank one case). The proof is based on the description of the derived category of $\mathcal{D}$-modules on X by a noncommutative version of the Beilinson transform on the projective line. The technique applies equally in higher dimensions, where the applications to integrable systems or representation theory are less obvious.

3.2. **Elliptic Representation Theory.** Much of the theory of integrable systems can be viewed as a “semiclassical limit” of representation theory of Lie groups and algebras, in which noncommutative algebras and their commutative subalgebras degenerate to Poisson algebras and commuting Hamiltonians. Our work in [BNe2, BNe5] can be viewed as relating meromorphic solitons of the KP and Toda hierarchies to the semiclassical geometry of moduli spaces of bundles on elliptic curves and their cuspidal and nodal degenerations.

The quantization of this theory leads us into the fascinating nascent subject of elliptic representation theory. Underlying this notion is a trichotomy (Lie algebras, Lie groups, elliptic algebras), which is realized equivalently in the classifications of curves of genus one (cuspidal, nodal, elliptic) or one–dimensional algebraic groups (additive, multiplicative, elliptic). The corresponding trichotomy (rational, trigonometric, elliptic) in the classification of integrable systems and quantum groups (which already manifested itself in the previous section) is at the center of a thriving algebraic discipline, developed by Cherednik, Felder, Etingof and others, see the ICM addresses [C, Fe, E]. Among the zoo of resulting algebraic structures are the double affine Hecke algebras, Macdonald polynomials, Sklyanin algebras, and elliptic quantum groups.

The approach to the representation theory of Lie groups described in Section 2 can be interpreted as embedding it in the geometric Langlands program on nodal curves of genus one. The program we have developed approaches elliptic representation theory as a deformation of classical representation theory corresponding to deforming from the nodal curve to smooth elliptic curves. This brings to bear on the unruly subject a unified perspective and the sophisticated technology of the geometric Langlands program. Perhaps the most interesting and challenging aspect of this program is the development of (finite, q- and elliptic) difference operator analogs of the fundamental role of differential operators in integrable systems and geometric representation theory. In particular we have been developing a difference version of the geometric Langlands conjecture, which should play the same role for quantum groups as the original conjecture plays for algebraic groups. The most approachable example of this general setting is the nodal case, where we obtain a Langlands classification for representations of double affine Hecke algebras, providing a proper setting for work of Baranovsky, Evens and Ginzburg [BEG]:

Conjecture 3.3. The category of modules over the double affine Hecke algebra is equivalent to a category of difference modules on the stack of semistable bundles on the nodal cubic curve, and the corresponding Langlands parametrization recovers the conjectural classification of [BEG].

In very recent work we have shown that the basic technical difficulties in the difference Langlands program can be handled using an extension of our ideas. This gives us confidence that the above conjecture, as well as its generalizations to the wide-open elliptic setting, is accessible with our techniques in the near future.

3.3. **Noncommutative geometry and Dijkgraaf-Vafa theory.** The best understood class of solutions to soliton hierarchies come from the Krichever construction. This roughly speaking assigns solutions to line bundles on algebraic curves (so-called spectral curves), so that the hierarchy corresponds to straight-line motions on the Jacobian. The Krichever class of solutions does not cover the important examples coming from matrix models and Gromov-Witten theory, for example the Kontsevich-Witten solution of KdV describing intersection theory on the moduli space of curves. However, these are examples of *isomonodromy solutions* of KP, which are associated instead to vector bundles with meromorphic connection on curves. The theory of isomonodromic deformation (which I studied with Frenkel using vertex algebras in [BF2]) is a nonlinear deformation of the theory of spectral curves for classical integrable systems. During our work on the KP hierarchy [BNe2] (and in particular [BNe6] relating $\mathcal{W}$-algebras and Wilson's adèlic Grassmannian [W]) we formulated a similar noncommutative-geometric ($\mathcal{D}$-module) theory of its isomonodromy solutions, giving a mathematical interpretation to the work of G. Moore on the string equation [M].

Our interest in isomonodromy equations and their generalizations has received a great new impetus thanks to exciting connections with physics. Dijkgraaf-Vafa theory (see in particular [ADKMV]) is a powerful but mysterious collection of ideas relating topological strings, matrix models, $\mathcal{W}$-symmetry and integrable hierarchies. Dijkgraaf has recently proposed [D] a general interpretation of Dijkgraaf-Vafa theory in terms of integrable systems built out of the noncommutative geometry of $\mathcal{D}$-modules on curves. The deformation parameter $\hbar$ interpolating between differential operators and their symbols (or between isomonodromic and isospectral systems) is identified with the genus expansion for the dual topological string (Gromov-Witten) theory. From my discussions with Dijkgraaf there has emerged a precise conjecture describing a general quantization of isospectral systems and their free fermion description, generalizing the descriptions of isomonodromy above and of the KP hierarchy in [BNe2]. A special case of this reads as follows:

Conjecture 3.4. A filtered holonomic $\mathcal{D}$-module on $\mathbb{P}^1$ defines a solution of a generalized KP hierarchy, with a set of times attached to each commutative point of the $\mathcal{D}$-module (considered as a noncommutative spectral curve).

I am confident that this new development will provide new applications and extensions of my work and perhaps a new avenue to uncovering the geometry behind the integrable structure of topological string theory.

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