PRACTICE EXAM FOR THE SECOND MIDTERM EXAM

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1. Notes

This practice exam has more problems than the real exam will. To make most effective use of this document, take the exam under conditions simulating the real exam — no book, no calculator.

(1) Short answer questions:
  (a) Explain when using the linear approximation is a good way to estimate change in a function.
  (b) Explain the relationship of the compound interest formula to the exponential function.
  (c) If we differentiate the equation $x^2 + y^2 = 1$ with respect to x, we get $2x+2yy' = 0$. If we differentiate with respect to t, we get $2xx'+2yy' = 0$. Why are these answers different?
  (d) Explain in words why the derivative is zero at a local maximum.

(2) Show how to derive the quotient rule from the product rule. (Hint: use the fact that $\frac{f(x)}{g(x)} = f(x)(g(x))^{-1}$.)

(3) Compute the following derivatives:
  (a) $f(x) = x^{x^2+3+4}$.
  (b) $f(x) = 19x^{56} + x^4 + \frac{x^2}{x^3+17}$.
  (c) $f(x) = x^2 \sin e^{x^3+\ln(\cos x^2)}$.
  (d) $f(x) = \frac{\sqrt{x^3+x+1}}{(x^4+x^8-12x^2+3)^4}$.
  (e) $f(x) = 10^{x^2+2}$.

(4) If \$1000 is invested at 4% interest, find how long it takes to double if we compound annually and continuously. Suppose we start with \$2000 — how long is the doubling time then?

(5) Find the normal line to the curve $y^2 = 3x^3 + x + 1$ at the point(s) where $x = 0$.

(6) Three people are running away from a sleeping bear. If Irving is running east at 10 mph, Maria is running west at 15 mph, and Olga is running north at 25 mph, find the rate of change of the sum of the distances between Irving and Olga and Irving and Maria after 15 minutes.

(7) The sides of a cube are expanding at a rate of 2 centimeters/ sec. Compute the rate of change of the volume and the surface area when the side length is 20 cm.

(8) Use linear approximation to estimate $\log_2 2.1$.

Date: October 13, 2011.

(9) Consider the linear approximation to $f(x) = x^2$ at $x = 0$. When estimating the change, when is the error in the linear approximation more than 50% of the actual value?

(10) Find the absolute minimum and absolute maximum of the following functions on the given interval:
   (a) $f(x) = xe^{\frac{-x^2}{8}}$ on [-1,4].
   (b) $f(x) = \frac{x}{x^2-x+1}$ on [0,3].
   (c) $f(x) = x^{\frac{1}{3}}(8 - x)$ on [0,8].

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