PRACTICE EXAM FOR FIRST MIDTERM EXAM

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1. Notes

This practice exam has more problems than the real exam will. To make most effective use of this document, take the exam under conditions simulating the real exam — no book, no calculator.

(1) Short answer questions:
   (a) Do all functions that have an inverse have a derivative?
   (b) Give an example of a function which is continuous but not differentiable.
   (c) Consider the function $f(x)$ defined to be 0 if $x \leq 0$ and 1 otherwise. Although $f(-1) = 0$ and $f(2) = 1$, f never takes on the value 0.5. Why doesn't this contradict the intermediate value theorem?
   (d) Explain how to use continuity to evaluate limits.
   (e) Explain the horizontal line test.

(2) Solve the following equations for x.
   (a)
   $$5^{3x} = 25^{x^2-x+2}.$$
   (b)
   $$2^{2x-2} = 7^{x^2}$$
   (c)
   $$e^{2x} - 4e^x + 3 = 0.$$

(3) Find the inverse of x^4 on the domain $[0, 64]$, on the domain $[-64, 64]$, and on the domain $[-64, 0]$.

(4) Find the inverses of the following functions algebraically:
   (a) $f(x) = 5 - e^{-x}$.
   (b) $f(x) = \frac{1+x}{1-x}$.
   (Please explain any restrictions on the domain or range.)

(5) Sketch the graph of the inverse of the function $y = \ln(x - 3)$.

(6) The position of a particle at time t is given by $f(t) = t^3 - t + 1$.
   (a) Find the average velocity over the interval $[0, 2]$.
   (b) Find the derivative using the definition, and find the average acceleration over the same interval.

(7) Describe the behavior of the slope of the tangents to $\sin x$ as x varies over $[0, 2\pi]$.

(8) Draw a graph of the position and a separate graph of the velocity of your car as you drive across the country. (Please assume realistic conditions; e.g., you need to sleep occasionally.)

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(9) Suppose that $\lim_{x\to 1}(f(x))^2 = 3$. What is $\lim_{x\to 1} f(x)$? (Does it have to exist?)

(10) Prove formally that $\lim_{x->\infty} 3e^x$ is ∞.

(11) Find the following limits:

(a)
$$\lim_{x\to -\infty} \frac{x^5+3}{(x^{15}+3x^9-4x^6+2)^{\frac{1}{3}}}.$$

(b)
$$\lim_{x\to 3} \frac{x^2-x-6}{x-3}.$$

(12) Use the intermediate value theorem to:

(a) Locate a solution for the equation:
$$\log_2(x+1)+\log_4(x-1)=1.$$

(b) Show that $x^4 = -1$ has no solutions.

(13) (a) Consider the following procedure: take a positive number x, round down to an integer, take the remainder when you divide by 12, add 1, and find the number of days in that month. Does this describe a continuous function?

(b) Where is $f(x) = e^{\sin(x)+x^{-10}\sqrt{x}}$ continuous?

(14) Let $f(x) = \frac{(x+3)^3}{(x-4)(x-1)(x-2)}$. Sketch the horizontal and vertical asymptotes of $f(x)$.

(15) Are the slopes of the tangents of $f(x) = x^2+2$ and $g(x) = 3x^3-4x+1$ ever parallel? (Please use the definition to find the derivatives in order to do this.)

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