PRACTICE EXAM #1 FOR THIRD MIDTERM EXAM

ANDREW J. BLUMBERG

1. Notes

This practice exam has more problems than the real exam will. To make most effective use of this document, take the exam under conditions simulating the real exam — no book, no calculator.

(1) Short answer questions:
   (a) Why doesn't L'Hopital's rule imply that
   $$\lim_{x\to 0}\frac{x+3}{x^2+3x+1}=\lim_{x\to 0}\frac{1}{2x+3}=\frac{1}{3}?$$
   (b) Please state the mean value theorem.
   (c) We developed the theory of integration using rectangles. Why didn't we use circles instead?

(2) Sketch the curve $\ln(x^2-3x+2)$ using derivative information.

(3) Compute the following limits:
   (a) $\lim_{x\to\infty} x\sin(\frac{1}{x})$.
   (b) $\lim_{x\to 0}\frac{x^2+2x+1}{3x^2-4x+5}$.
   (c) $\lim_{x\to\infty}\frac{\ln(\sqrt{x})}{x^2}$.

(4) Find the following derivatives.
   (a) $f(x)=\int_3^{x^2}\ln y dy$.
   (b) $f(x)=\int_5^{\ln(x^2+x+1)}e^z dz$.

(5) A rope which is 20 feet long is cut into two pieces; one piece is used to make a circle, and one piece is used to make a square. How should the rope be cut in order to maximize the area enclosed? To minimize the area enclosed?

(6) For the function $f(x)=e^{-x^2}$,
   (a) Approximate the definite integral $\int_{-1}^4 f(x)dx$ using a Riemann sum with 5 intervals and using the lefthand side of the rectangle.
   (b) Approximate the definite integral $\int_{-1}^4 f(x)dx$ using a Riemann sum with 5 intervals and using the midpoint of the rectangle.
   (c) Write the definite integral as a limit expression.
   (d) Can you use substitution to find an antiderivative for this function?

(7) Compute the following definite integrals:
   (a) $\int_{-3}^3 x\sqrt{x-1}dx$.
   (b) $\int_0^{10}\frac{e^{\frac{1}{x}}}{x^2}dx$.
   (c) $\int_{-5}^0(x\sin(3x^2+4)+2^x)dx$.

Date: November 4, 2011.

(8) Suppose that $-1 \leq f(x) \leq 1$. What can we say about $\int_{-3}^{3} f(x)dx$?

E-mail address: blumberg@math.utexas.edu