PRACTICE EXAM #2 FOR THIRD MIDTERM EXAM

ANDREW J. BLUMBERG

1. NOTES

This practice exam has more problems than the real exam will. To make most effective use of this document, take the exam under conditions simulating the real exam — no book, no calculator.

(1) Short answer questions:
   (a) Suppose that $f'(x) = g'(x)$. What can we say about f and g?
   (b) Define the definite integral $\int_a^b f(x)dx$.
   (c) Explain what the second derivative tells us about maxima and minima.
   (d) Riemann sums can be computed using the left endpoint, the midpoint, and the right endpoint. Which is better? Explain.

(2) Sketch the curve $f(x) = \frac{e^x}{1-e^x}$ by finding maxima, minima, concavity, inflection points, and so forth.

(3) Define the function $f(x) = \frac{e^x - 1}{x}$ for $x \neq 0$, and $f(0) = 1$. Compute $f'(0)$ using L'Hopital's rule. (Hint: use the definition of the derivative.)

(4) Find the derivative of

$$\int_{-x^2}^{x^2} \ln(\sin x)dx.$$

(Hint: split this up!)

(5) Solve the following optimization problems:
   (a) Find the area of the rectange of maximal area which has two vertices on the x-axis and two vertices with $x > 0$ and lying on the curve $9 - x^2$.
   (b) I want to make a cylindrical barrel to hold 10 cubic meters of molasses. Assume that the sides cost 36 per square meter and the (circular) ends cost 18 per square meter. What should the dimensions of the cheapest barrel with the required volume be?

(6) Let $f(x) = \frac{1}{x}$.
   (a) Approximate the definite integral $\int_1^5 f(x)dx$ using a Riemann sum with 2 intervals and using the lefthand side of the rectangle.
   (b) Approximate the definite integral $\int_1^5 f(x)dx$ using a Riemann sum with 4 intervals and using the lefthand side of the interval.
   (c) Compute the definite integral using the fundamental theorem of calculus.

(7) Compute the following definite integrals.
   (a) $\int_0^{100} 8x^2 e^{x^3 + 5}dx$.
   (b) $\int_{-1}^1 e^x \sin(e^x)dx$.

Date: November 7, 2011.

E-mail address: blumberg@math.utexas.edu