Homework Solutions
Chapter 16

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51. From Mathematical Insight 16.1, the critical mass needed for a cloud to collapse is
$$M_{\text{balance}} = 18 M_\odot \sqrt{\frac{T^3}{n}}.$$
We are told that the temperature, T, is 200 K and the density, n, is 300,000 molecules per cubic centimeter, so we can find the critical mass: $93 M_\odot$. So stars from the first generation were probably around 93 times the mass of the Sun or more. Looking it up in Chapter 15, we see that such stars have lifetimes of less than 10 million years.

52. (a) We will take the mass density of the Sun and then divide by mass per particle to get the average number density of particles in the Sun. First, we need the density of the Sun. For this, we need the mass ($2 \cdot 10^{33}$ g) and the volume. We compute the volume by assuming that the Sun is a sphere and using
$$\text{Volume}_{\text{sphere}} = \frac{4}{3}\pi r^3.$$
The radius of the Sun is $6.96 \cdot 10^{10}$ cm, so the volume of the Sun is $1.41 \cdot 10^{33}\,\text{cm}^3$. Density is mass over volume, so the average density of the Sun is $1.42\,\text{g/cm}^3$. Finally, we just need to convert this to number density by dividing the mass density by the mass per particle. The mass per particle is given as 10^{-24} g, so we get the average number density of the Sun to be $1.42 \cdot 10^{24}$ particles/cm^3.

(b) from Mathematical Insight 16.1, the critical mass needed for a cloud to collapse is
$$M_{\text{balance}} = 18 M_\odot \sqrt{\frac{T^3}{n}}.$$

Solving for the temperature gives us

$$T = \sqrt[3]{n\left(\frac{M_{\text{balance}}}{18M_\odot}\right)^2}.$$

We know the Sun is in balance, so M_{balance} must be the mass of the Sun ($1M_\odot$). We found the number density inside the Sun in part (a). Calculating, we find that the Sun must be 16 million K.

(c) The Sun's core is actually around 15 million K, but this estimate is quite close.

54. (a) Newton's version of Kepler's third law states that the period, p, the orbital separation, a, and the total mass, M, are related by the formula

$$a^3 = \frac{GM}{4\pi^2}p^2.$$

We are told that the total mass is twice the mass of the Sun. The mass of the Sun is $2 \cdot 10^{30}$ kg, so the total mass in this system is $4 \cdot 10^{30}$ kg. The period is 10 days, which we need to convert to seconds: $8.64 \cdot 10^5$ seconds. We can therefore find the orbital separation by plugging in the numbers. We get $1.72 \cdot 10^{10}$ m.

(b) The average speed will be the distance that the planets travel around their orbits over the period. We know the period (in both days and seconds), so we just need the average distance that the stars each travel. If we assume that the orbits are circular, we can take the distance traveled to be the circumference of a circle with a diameter equal to the separation found in part (a). The circumference is πa. Using the separation in part (a), this tells us that each star travels $5.40 \cdot 10^{10}$ m in each orbit. Dividing by the period gives us the average speed: 62,500 meters per second.

(c) Angular momentum is mass $\cdot$ speed $\cdot$ radius. We have all three quantities now: the mass of each star is $2 \cdot 10^{30}$ kg, the speed is 62,500 meters per second, and the radius is half the separation, $8.6 \cdot 10^9$ m. Calculating the angular momentum, we obtain $1.08 \cdot 10^{45}$ kg $\cdot$ m^2/sec.

(d) Let us assume that all of the Sun's mass was located a solar radius from the Sun's rotation axis. In this case, the angular momentum will be mass $\cdot$ speed $\cdot$ radius, where the mass is the mass of the Sun and the radius is the radius of the Sun ($6.96 \cdot 10^8$ m). We

have the angular momentum from part (c), so we can solve for the speed at which the Sun's surface would have to move:

$$\begin{aligned}\text{speed} &= \frac{\text{angular momentum}}{\text{mass} \cdot \text{radius}} \\ &= \frac{2.16 \cdot 10^{45}\ \frac{\text{kg}\cdot\text{m}^2}{\text{sec}}}{(2 \cdot 10^{30}\,\text{kg}) \cdot (6.96 \cdot 10^{8}\,\text{m})} \\ &\approx 1,552,000\,\frac{\text{m}}{\text{sec}}.\end{aligned}$$

We estimate that the Sun's surface would have to move at 77,600 meters per second to have the same angular momentum as each of the two stars.

(e) The escape velocity from the Sun's surface is

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}.$$

Substituting the values for the Sun, we get an escape velocity of $619,000\,\text{m/s}$. Our estimate of the speed that the material would need in order to have the same angular momentum as the two orbiting stars is more than twice this value. So if the Sun were spinning this fast, the material would escape off the surface and the Sun would fall apart.