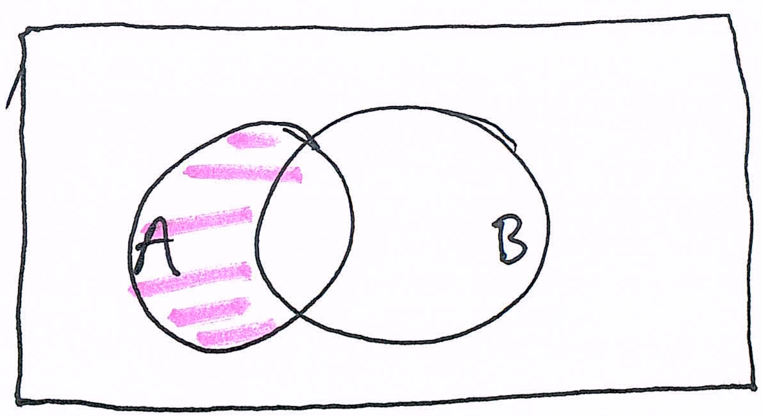
Set Theory (contd.)

4. Difference. $A \setminus B$ is the set of objects in $A$ but not in $B$.

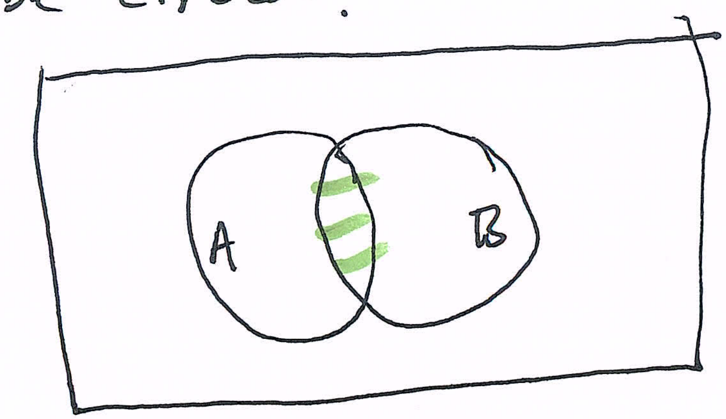
$A = \{2, 4, 6, 8, 10, 12\}$ $B = \{3, 6, 9, 12\}$.

$A \setminus B = \{2, 4, 8, 10\}$.

Venn Diagrams : Universal set is a rectangle. The sets of interest will be circles.




$A \setminus B$




$A \cap B$

Axioms of Probability. Given a sample space $S$ (universal set), each event $E$ (subset of $S$) is assigned a probability $pr(E)$ satisfying

1. $0 \le pr(E) \le 1$
2. ~~pr(S) = 1~~ $pr(S) = 1$
3. If $E_i$, $i \in I$, $I$ an index set, are mutually exclusive ($E_i \cap E_j = \phi$ if $i \ne j$) then

$$pr\left(\bigcup_{i \in I} E_i\right) = \sum_{i \in I} pr(E_i)$$

$E_1 = \{2, 3, 4, 6\}$ $E_2 \{8, 9, 10, 12\}$

$$pr(E_1) = \frac{N(E_1)}{N(S)} = \frac{4}{12} \qquad pr(E_2) = \frac{N(E_2)}{N(S)} = \frac{4}{12}$$

$pr(E_1 \cup E_2) = \frac{8}{12}$ $\quad E_1 \cup E_2 = \{2,3,4,6,8,10,9,12\}$.

Basic Rules of Probability.

1. Negation Rule: $Pr(E^c) = 1 - pr(E)$

2. Inclusion - Exclusion Principle. $E, F$ Two events

$$Pr(E) + Pr(F) = Pr(E \cup F) + Pr(E \cap F).$$

$$Pr(E \cup F) = pr(E) + pr(F) - pr(E \cap F)$$

Example: $A = \{2,4,6,8,10,12\}$ $B = \{3,6,9,12\}$.

$A \cap B = \{6,12\}$. $A \cup B = \{2,3,4,6,8,9,10,12\}$

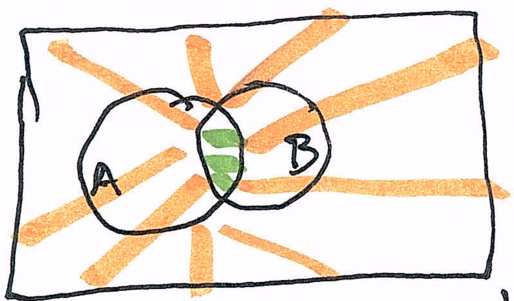
$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B).$$

$Pr(A) = \frac{6}{12} = \frac{1}{2}$ $\quad Pr(B) = \frac{4}{12} = \frac{1}{3}$ $\quad Pr(A \cap B) = \frac{2}{12} = \frac{1}{6}$

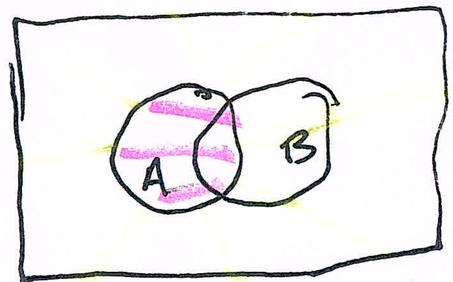
$Pr(A \cup B) = \frac{8}{12} = \frac{2}{3}$ $\quad = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}.$

DeMorgan's Laws: For any sets $A, B$.

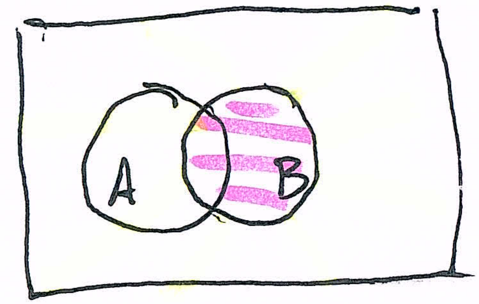
$(A \cap B)' = A' \cup B' \qquad (A \cup B)' = A' \cap B'$




$(A \cap B)'$




$A'$




$B'$

Venn Box Diagram

| | $A$ | $A'$ | |
|---|---|---|---|
| $B$ | $pr(A \cap B)$ | $pr(A' \cap B)$ | $pr(B)$ |
| $B'$ | $pr(A \cap B')$ | $pr(A' \cap B')$ | $pr(B')$ |
| | $pr(A)$ | $pr(A')$ | 1 |

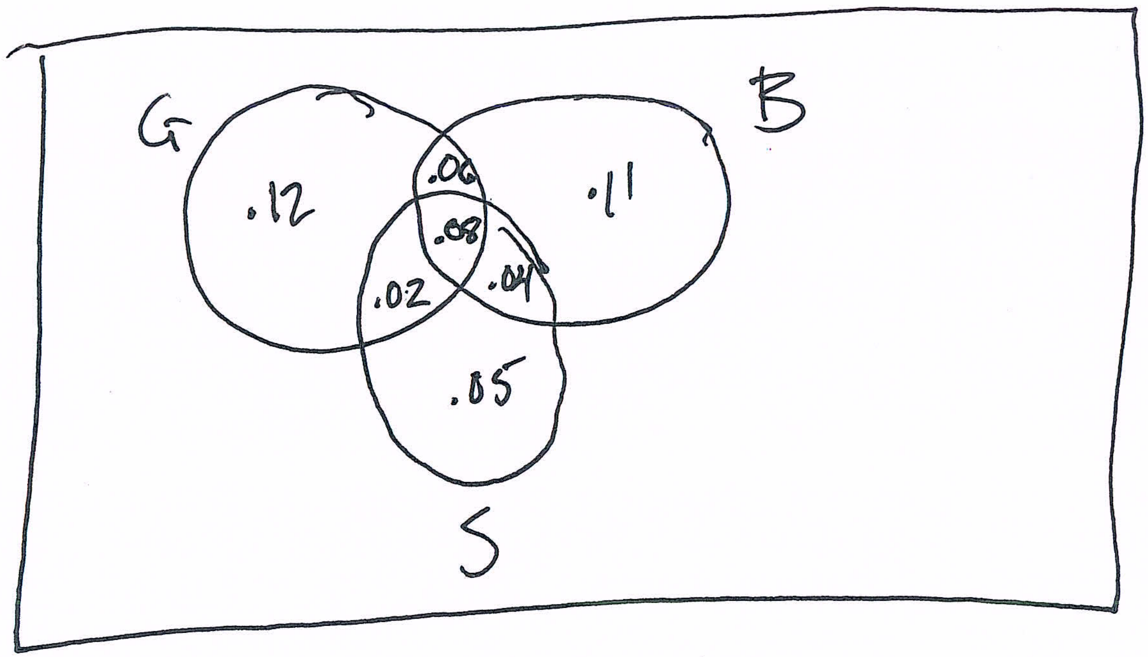
Example: 90% of residents of the US or Canada are citizens of US or Canada. 40% of Canadians are dual US nationals. 10% of residents of US or Canada are citizens of Canada.

C - citizens of Canada

A - citizens of US

| | $A$ | $A'$ | |
|---|---|---|---|
| $C$ | .04 | .06 | .1 |
| $C'$ | .8 | .10 | .9 |
| | .84 | .16 | 1.0 |

# Mickey Mouse Diagram




28% Gymnastics

29% Baseball

19% Soccer

14% G+B

12% B+S

10% G+S

8% G+B+S