Generalized Symmetries, Anomalies, and Observables

Aspen Center for Physics

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Topics: Generalized symmetries in quantum field theory and condensed matter physics
Anomalies of ordinary and generalized symmetries
Observables and symmetries

Scientific Description: Since the late '80s there have been several workshops and working groups at the ACP which have brought together physicists and mathematicians around topics in high energy theory and, more recently, condensed matter theory. The most recent workshop, *Boundaries and Defects in Quantum Field Theories*, was held in 2016. The current proposal combines cutting edge ideas in particle physics and condensed matter physics with ideas in mathematics. The topics are unified by the notion of *higher symmetries*. In the past few years these have played a central role in many field theoretic investigations in high energy theory and condensed matter. Our workshop will catalyze further developments by bringing together physicists and mathematicians who are already working on these topics and others with the expertise to do so. There is a broad spectrum of ideas represented here, and this justifies the 4-week length we propose.

In Quantum Field Theory, there exist local and non-local observables. An example of a local observable is a detector measuring the flux of particles through some small region of space. A non-local observable could be the number of ground states of the theory. Similarly, one can define different notions of symmetry. We can have symmetries that act only on local observables and local excitations, and we may have symmetries that also act on non-local operators and non-local excitations. Traditionally, the applications and study of symmetries has been almost exclusively restricted to the former kind. Nowadays it is customary to refer to symmetries that act on local observables as "zero-form symmetries" since their parameters are either constants or, when gauged, functions on space-time. However, symmetries that act only on non-local excitations also occur. For example, pure Yang-Mills theory has physical string excitations (which have been observed on the lattice) which are similar to the Abrikosov strings in superconductors. These strings, when wrapped around non-contractible curves, cannot decay to glueballs. One can say that Yang-Mills theory has a symmetry that acts on these strings. Symmetries that act on strings are called "one-form symmetries." Similarly, conservation laws for higher-dimensional extended objects lead to "higher-form symmetries."

Higher-form symmetries appear in many situations. Like zero-form symmetries they may be spontaneously broken, they may be softly broken, they may have 't Hooft anomalies, and they can be coupled to background gauge fields. Furthermore, there is an important interplay between one-form symmetries and zero-form symmetries: if we compactify a quantum field theory with a one-form symmetry on a circle, then we will get at low energies an effective theory in one dimension lower with a zero-form symmetry (as well as a one-form symmetry). Physically, such a circle compactification may correspond to studying the model at finite temperature in equilibrium or a non-thermal compactification as a function of radius. In the latter phase transition may be avoided with the judicious choice of matter content and boundary conditions, as in the supersymmetric theories with supersymmetry preserving boundary conditions as well as in some special non-supersymmetric QCD-like theories. Therefore, the study of one-form symmetries also sheds important light on the behavior of interesting quantum field theories at finite temperature or on some nontrivial spatial manifolds. Even if one is studying a model that has no exact higher-form symmetries, these symmetries might be approximate. For example, in $SU(N)$ QCD with fundamental dynamical quarks, the strings can decay and there is no one-form symmetry. However, if the quarks are heavy, or in a higher-dimensional representation, this decay is very improbable (or even forbidden, depending on the representation), and hence the one-form symmetry is still important. Similarly, in condensed matter systems, one often starts from a local lattice Hamiltonian without gauge fields. Such models would not have a one-form symmetry. However, these models often develop emergent gauge fields at long distances and accidental one-form symmetries may appear. The existence of one-form symmetries, even if accidental, severely constrains the dynamics.

Since the notion of higher-form symmetry in quantum field theory was proposed in 2014, there have been many diverse applications of this concept. Using such symmetries it is possible to prove that some quantum field theories cannot flow to trivial gapped phases. In many cases, using the 't Hooft anomalies of such one-form symmetries, it is possible to determine the infrared phases of interesting models in 2+1 and 3+1 dimensions. There are also many applications to the dynamics of supersymmetric theories. One-form symmetries were successfully used to constrain the thermal phase diagram of Yang-Mills theory and of similar related models in lower dimensions. The study of one-form symmetries and their anomalies has provided striking new tests of dualities. Higher-form symmetry turned out essential for understanding bosonization in more than one spatial dimension. It has been shown that a bosonic dual of a fermion theory in d space-time dimensions must have a $(d-2)$-form Z_2 symmetry with a particular 't Hooft anomaly. This provides a consistency check on various fermion-boson dualities proposed in the literature. Other interesting potential examples of generalized symmetries with 't Hooft anomalies are models which realize fracton phases of matter. Farther afield, one-form symmetries have been used to formulate the hydrodynamic theory of strongly interacting plasmas.

Our discussion so far has focused on generalized symmetry *groups*. In recent mathematical investigations of symmetry structures in quantum field theory a significant development is the theory of factorization *algebras*. They are emerging as a central topic. Factorization algebras in the topological setting coincide with algebras over little disc operads, an algebraic tool introduced by topologists to detect higher groups, in the guise of iterated loop spaces. They also also provide a new method to construct topological field theories via factorization homology, following work of Lurie, Ayala-Francis, and others. In the conformal setting, as pioneered by Beilinson-Drinfeld, factorization algebras provide a powerful new geometric perspective on the theory of vertex algebras, which has re-emerged recently to play a prominent role in the study of supersymmetric gauge theory; see in particular work of Beem, Rastelli and collaborators and Gaiotto in physics, and of Arakawa in mathematics. Finally factorization algebras can be used to capture the structure of operator product expansion of operators in general quantum field theory, as developed by Costello-Gwilliam, and, through the mechanism of factorization homology, to encode constraints—Ward identities—arising from symmetries of a theory. They provide a powerful language to express subtle aspects of quantum field theory mathematically and thus a fertile area for interactions between mathematicians and physicists.

Symmetry is always at the forefront of theoretical models, and in the past few years these novel variations—higher groups, factorization algebras—are yielding direct physical consequences, as outlined above. The commonalities among high energy theorists, condensed matter theorists, and mathematicians working in these areas provides a solid basis for interaction; the differences in approach inspires confidence that a summer workshop at ACP will catalyze further progress and open up new avenues for investigation.