QFT and Geometric Bordism Categories

The reasonable effectiveness of a mathematical definition

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What is Quantum Field Theory?

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The definition of conformal field theory

Graeme Segal

I shall propose a definition of 2-dimensional conformal field theory which I believe is equivalent to that used by Friedan et al. The idea arises from joint work with Quillen.

§1 The definition

The category $\mathcal{C}$ is defin

objects $\{C_n\}_{n\geq 0}$, where C_n

TOPOLOGICAL QUANTUM FIELD THEORIES

by MICHAEL ATIYAH

To René Thom on his 65th birthday.

We come now to the promised axioms. A topological quantum field theory (QFT), in dimension d defined over a ground ring Λ, consists of the following data:

(A) A finitely generated Λ-module $Z(\Sigma)$ associated to each oriented closed smooth d-dimensional manifold Σ,

(B) An element $Z(M) \in Z(\partial M)$ associated to each oriented smooth $(d+1)$-dimensional manifold (with boundary) M.

These data are subject to the following axioms, which we state briefly and expand upon

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Plan of Lecture: Two applications + questions

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Plan of Lecture: Two applications + questions

- Invertible topological theories and phases of matter
- Line operators in 4-dimensional gauge theory
- Formulate issues for scale-dependent theories

QFT as a Representation of Geometric Bordism

Definition: A *field theory* is a homomorphism ($\otimes$-functor)

$$F\colon \mathrm{Bord}_{\langle n-1,n\rangle}(\mathcal{F}) \longrightarrow \mathrm{Vect}_{\mathbb{C}}^{\mathrm{top}}$$

$\amalg$ $\qquad\qquad$ $\otimes$

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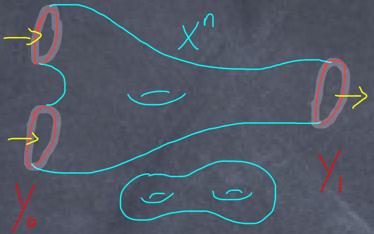
$\mathcal{F}$	sheaf of background fields (orientation, metric, . . .)
$\mathrm{Vect}_{\mathbb{C}}^{\mathrm{top}}$	category of complex topological vector spaces

$\mathrm{Bord}_{\langle n-1,n\rangle}(\mathcal{F})$:

objects



morphisms

$X : Y_0 \longrightarrow Y_1$

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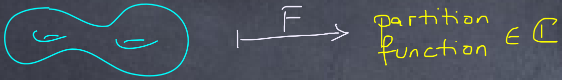
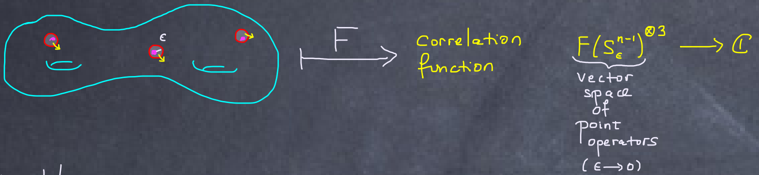




Wick rotated!

QFT as a Representation of Geometric Bordism

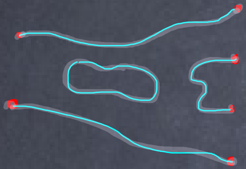
Definition: A *fully extended field theory* is a homomorphism

$$F\colon \operatorname{Bord}_n(\mathcal{F}) \longrightarrow \mathcal{C}$$

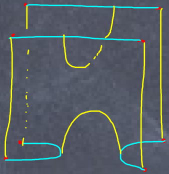
$\mathcal{F}$	sheaf of background fields (orientation, metric, . . .)
$\mathcal{C}$	topological n-category

$\operatorname{Bord}_n(\mathcal{F})$:

objects



1-morphisms



2-morphisms

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Longrange Effective TFT of a Gapped System

Low energy behavior: energy gap $\Longrightarrow$ topological field theory α

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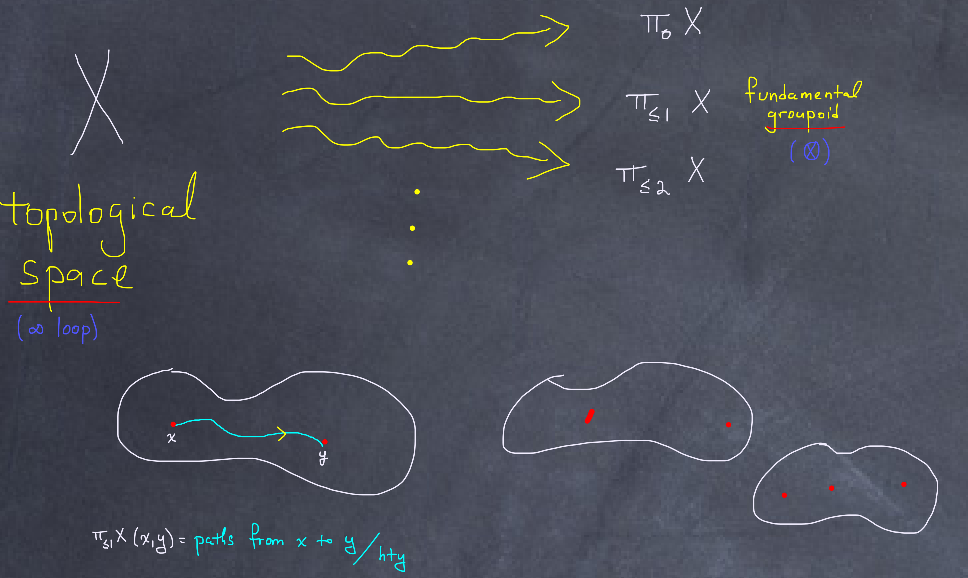
Invertible topological field theories are maps of infinite loop spaces:

$$\begin{array}{ccc} \mathrm{Bord}_n(\mathcal{F}) & \xrightarrow{\ \alpha\ } & \mathcal{C} \\ \downarrow & & \uparrow \\ |\,\mathrm{Bord}_n(\mathcal{F})| & \xrightarrow{\ \tilde{\alpha}\ } & \mathcal{C}^{\times} \end{array}$$

Classification of Invertible Topological Field Theories



Classification of Invertible Topological Field Theories

Thm (Galatius-Madsen-Tillmann-Weiss): $|\,\mathrm{Bord}_n\,| \simeq \Sigma^{\infty+n} MTO_n$

$$MTO_n = \mathrm{Thom}(-\xi_n \longrightarrow BO_n)$$

$$(\Sigma^1 MTO_1 \longrightarrow \Sigma^2 MTO_2 \longrightarrow \Sigma^3 MTO_3 \longrightarrow \cdots) \longrightarrow MO$$

Thom's bordism spectrum

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Many explicit computations (`arXiv:1406.7278`)

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Thm (F.-Hopkins): The abelian group of 4d unitary invertible topological field theories/deformation with time-reversal (T) is:

(i) $(T^2 = (-1)^F)$ $\quad [\Sigma^4 MT\mathrm{Pin}_4^+, \Sigma^4 I\mathbb{C}^\times]_{\mathrm{unitary}} \cong \mathbb{Z}/16\mathbb{Z}$

(ii) $(T^2 = +1)$ $\quad [\Sigma^4 MT\mathrm{Pin}_4^-, \Sigma^4 I\mathbb{C}^\times]_{\mathrm{unitary}} \cong \mathbb{Z}/2\mathbb{Z}$

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Takeaway: The geometric bordism definition of extended field theory enables these computations.

Towards 2nd Application: Relative Field Theories

Warmup: σ-model into manifold M with fundamental group $\pi = \pi_1 M$

$$\pi \longrightarrow \widetilde{M} \longrightarrow M$$

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maps to $\tilde{M}$ — maps to M — Galois covers with group π

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Up one level: $A \to G \to \overline{G}$ covering of compact Lie groups, A finite

e.g. $\{\pm 1\} \longrightarrow SU_2 \longrightarrow SO_3$

$\{\mu^i\}_{i=1}^{n} \longrightarrow SU_n \longrightarrow PU_n$

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G-connections

$\overline{G}$-connections

A-gerbes [classified by $H^2(-;A)$]

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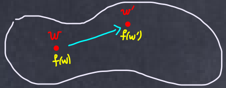
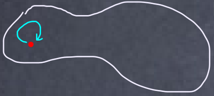
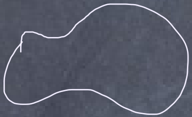
Integrate over the fibers to define a relative theory f







$f(w) = f(w')$

$B^2 A(X^n)$

$\pi_0 = H^2(X;A)$

$\pi_1 = H^1(X;A)$

$\pi_2 = H^0(X;A)$

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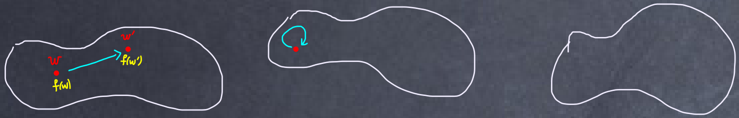

$$\pi_0 = H^2(X;A) \ni m$$

$$f(X) = \{ f(X)_m \}$$

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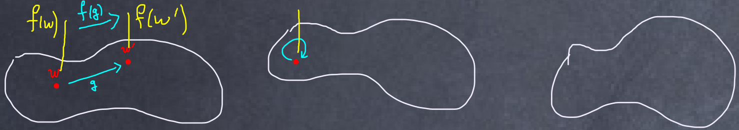

$$B^2 A(Y^{n-1})$$

$$\pi_0 = H^2(Y;A)$$
$$\pi_1 = H^1(Y;A)$$
$$\pi_2 = H^0(Y;A)$$

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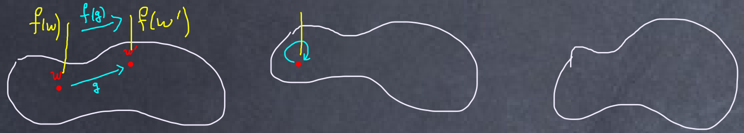

$$f(Y) = \{ \mathcal{H}_{m,e} \}$$

$$\pi_0 = H^2(Y; A) \ni m$$

$$\pi_1^\vee = H^1(Y; A)^\vee \ni e$$

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Definition: Let α be an extended $(n+1)$-dimensional quantum field theory. A field theory f relative to α is a homomorphism

$$f\colon \mathbf{1} \longrightarrow \tau_{\le n}\alpha$$

or

$$\widetilde{f}\colon \tau_{\le n}\alpha \longrightarrow \mathbf{1}$$

(see F.-Teleman `arXiv:1212.1692`)

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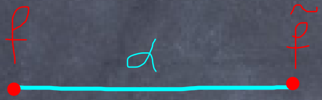
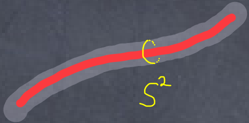
Two boundary conditions:



Line Operators in 4d Gauge Theories

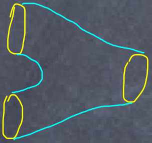


Point ("local") operators $F(S^3)$

↗ vector space

Line operators $F(S^2)$

↗ 1-category

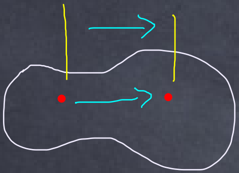


fusion of line operators

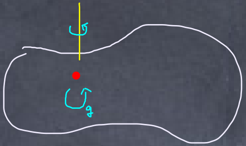
Line Operators in 4d Gauge Theories

For $f\colon \mathbf{1} \to \tau_{\leq 4}\alpha$ the 1-category of line operators is organized by the 2-category $\alpha(S^2)$. For α the topological theory of A-gerbes we find

$$f(S^2) = \{\mathcal{L}_{m,e}\} \qquad m \in H^2(S^2;A) \cong A \qquad e \in H^0(S^2;A)^\vee \cong A^\vee$$



$B^2A(S^2)$



$\pi_0 = H^2(S^2;A) \cong A \ni m$

$\pi_1 = H^1(S^2;A) = 0$

$\pi_2 = H^0(S^2;A) \cong A \ni g$

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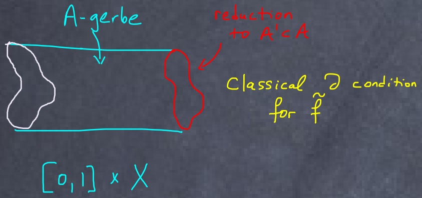
Fix (A', q) to define $\widetilde{f}\colon \tau_{\leq 4}\alpha \to 1$ and absolute theory $F = \widetilde{f} \circ f$

$$A' \leq A, \quad q\colon A' \to \mathbb{Q}/\mathbb{Z} \ \text{(quadratic)} \quad \Longrightarrow \quad b = e^{2\pi i\,\partial q}\colon A' \times A' \to \mathbb{C}^\times \ \text{(bilinear)}$$

F is a gauge theory with gauge group G/A'. (Recall covering $G \xrightarrow{A} \overline{G}$)



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Main assertion: A "higher Gauss law" implies

$$f(S^2) = \bigoplus_{\substack{m' \in A' \\ e \in A^\vee \text{ such that } e|_{A'} = b(m')^{-1}}} \mathcal{L}_{m',e}$$

$$H^2(S^2; A') \times H^0(S^2; A') \xrightarrow{b\,\smile} H^2(S^2; \mathbb{C}^\times) \xrightarrow{[S^2]} \mathbb{C}^\times$$

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For $f\colon \mathbf{1} \to \tau_{\leq 4}\alpha$ the 1-category of line operators is organized by the 2-category $\alpha(S^2)$. For α the topological theory of A-gerbes we find

$$f(S^2) = \{\mathcal{L}_{m,e}\} \qquad m \in H^2(S^2;A) \cong A \qquad e \in H^0(S^2;A)^\vee \cong A^\vee$$

Fix (A', q) to define $\widetilde{f}\colon \tau_{\leq 4}\alpha \to 1$ and absolute theory $F = \widetilde{f} \circ f$

$$A' \leq A, \quad q\colon A' \to \mathbb{Q}/\mathbb{Z} \text{ (quadratic)} \quad \Longrightarrow \quad b = e^{2\pi i\,\partial q}\colon A' \times A' \to \mathbb{C}^\times \text{ (bilinear)}$$

F is a gauge theory with gauge group G/A'. (Recall covering $G \xrightarrow{A} \overline{G}$)

Main assertion: A "higher Gauss law" implies

$$f(S^2) = \bigoplus_{\substack{m' \in A' \\ e \in A^\vee \text{ such that } e|_{A'} = b(m')^{-1}}} \mathcal{L}_{m',e}$$

Aharony-Seiberg-Tachikawa (`arXiv:1305.0318`)
Gaiotto-Kapustin-Seiberg-Willett (`arXiv:1412.5148`)

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- Define mass gap. Construct infrared topological theory.
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- Construct quantum moduli space. Infrared conformal theory.
- Reconstruction: field theory on Minkowski spacetime.