Problem Set # 10

M392C: K-theory

1. Apply the Weyl character formula to deduce the characters of some representations, for example some mentioned in the previous homework. Explore the rank 2 groups SO_4, Spin_4, SO_5, Spin_5, Sp_2 and U_2. Learn the graphic algorithm at the end of the article of Bott (see web page) for computing the character. There is graph paper available on that web page for SU_2. I welcome graph paper for other groups!

2. Let G be a compact Lie group. There is an infinite dimensional affine space of covariant derivatives on the tangent bundle which contains three special points: the covariant derivative from global parallelism by left translation, the covariant derivative from global parallelism by right translation, and the Levi-Civita covariant derivative of any bi-invariant Riemannian metric. Prove that these lie on an affine line, and more precisely the latter is the midpoint of the first two. The covariant derivative which appears in the Dirac family lies 1/3 the way from one endpoint to the other. Unknown: Is there a direct differential-geometric meaning of that covariant derivative?

3. Let V be a finite dimensional real vector space with inner product Q and $C(V,Q)$ the corresponding Clifford algebra. Recall that the spin group of V embeds in the group of invertible elements in $C(V,Q)$, so by differentiating the Lie algebra of the spin group also embeds in $C(V,Q)$. That Lie algebra is $\mathfrak{o}(V)$, the orthogonal Lie algebra of V. Derive a formula for that embedding. Write it explicitly if $\{e_a\}$ is an orthonormal basis of V.