REMARK ON SEPTEMBER 12 LECTURE

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The construction I gave in the proof of Theorem 2 in lecture, to deformation retract the λ-handle to the λ-cell, was wrong: the limit of the flow as $t \to \infty$ is not a continuous map. The argument can be made simply and more explictly as follows.

Recall the setup. We have a Morse coordinate system $x = (x^1, \dots, x^n)\colon U \to \mathbb{A}^n$ in which

$$f = -\left\{(x^1)^2 + \cdots + (x^\lambda)^2\right\} + \left\{(x^{\lambda+1})^2 + \cdots + (x^n)^2)\right\}. \tag{1}$$

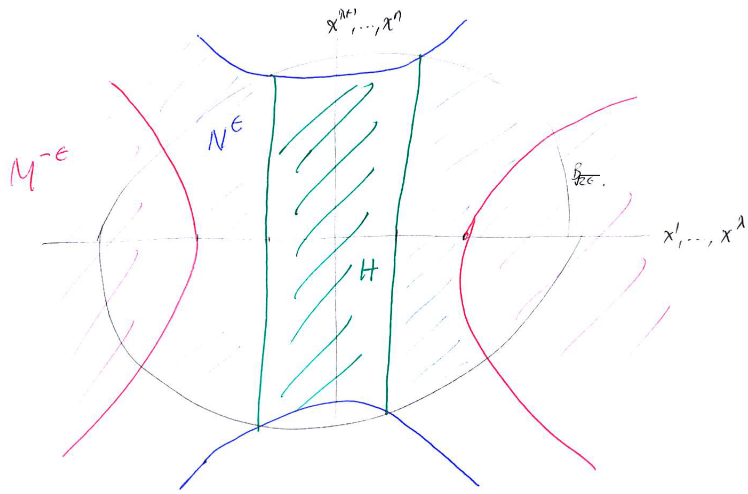

Figure 1. Critical point in Morse coordinates

The various regions are

$$\begin{aligned} M^\epsilon &= \{q \in M : f(q) \leqslant \epsilon\} \\ H &= \left\{q \in M^\epsilon \cap U : x^1(q)^2 + \cdots + (x^\lambda)^2 \leqslant \epsilon/2\right\} \\ N^\epsilon &= \overline{M^\epsilon \setminus H} \\ D^\lambda &= \{(x^1, \dots, x^n) \in U : x^1(q)^2 + \cdots + (x^\lambda)^2 \leqslant \epsilon/2,\ x^{\lambda+1} = \cdots = x^n = 0\} \end{aligned} \tag{2}$$

We construct a smooth deformation retraction of $M^\epsilon = N^\epsilon \cup H$ onto $N^\epsilon \cup D^\lambda$. Write $u = (u^1, \dots, u^\lambda)$, $v = (v^1, \dots, v^{n-\lambda})$, and $x = (u, v)$. Construct a smooth function $\rho\colon [0,1] \times \mathbb{R}^{\geqslant 0} \to \mathbb{R}^{\geqslant 0}$,

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written $\rho(t,r)=\rho_t(r)$, such that

$$\rho_t(r)=\begin{cases}1-t, & r\leqslant \epsilon/2;\\ 1, & r\geqslant \epsilon,\end{cases}$$
$$1-t\leqslant \rho_t(r)\leqslant 1. \tag{3}$$

The deformation retraction is defined by the formula

$$\varphi_t(u,v)=\bigl(\rho_t(|u|^2)u,v\bigr). \tag{4}$$