

Optimal Geometry in Nature, Art, and Mathematics

TC 310: Modes of Reasoning — Unique Number 42970

Fall 2012

“Equations are just the boring part of mathematics. I attempt to see things in terms of geometry.” — Stephen Hawking

Course description

In many cases, nature finds optimal solutions to geometric problems. For example, among all possible shapes spanning a loop of wire, a soap film always has least area. The hexagonal packing system used by bees in their honeycombs is the most efficient (least perimeter) way to divide a plane into equal areas. Remarkably, these and other shapes found in physical or biological systems also appear in abstract mathematics as solutions of certain optimization problems. Several of these forms also frequently appear in art and architecture, and are regarded as beautiful by many cultures.

These phenomena raise several interesting questions. What does it mean to have a “best” form? Indeed, is the concept of “best” even well defined in this context? Can mathematical reasoning prove that something is optimal in a precise sense? What roles do evolution and physical principles play in finding such forms? How and why do various cultures integrate such forms into their patterns of artistic expression? What motivates artists and architects to incorporate geometric principles in their designs?

Historically, scholars studied optimality in nature using both empirical and synthetic (philosophical or theological) approaches. Ultimately, the discussion of why these optimal forms occur in nature is properly one of philosophy, not mathematics. What contribution can mathematicians make to this discussion? Simply this: mathematics provides tools by which we can find rigorous meaning in what might otherwise be fuzzy concepts of what are “better” or “best” geometries.

The main focus of this course will investigate what insights can be gained by looking at nature and art through the lens of mathematics — specifically, by investigating how our ways of understanding mathematics and the natural world inform and interact with each other. In accord with the *Modes of Reasoning* rubric, we will devote particular attention to the roles of geometric reasoning and mathematical rigor in making our explanations intellectually rigorous. We will do this through four case studies, organized around the following topics and questions:

- (1) *Optimality and Modeling*: how do we model and find optimal forms?
- (2) *Symmetry and Transformations*: how do we recognize and compare symmetries?
- (3) *Mathematical Epistemology*: what do mathematicians accept as evidence and proof?
- (4) *Pattern and Abstraction*: how do we analyze repeating patterns in nature and art?

Basic information

Class location: GEA 127

Class times: 9:30–11:00 Tuesdays and Thursdays

Instructor: Associate Professor Dan Knopf

Email: danknopf@math.utexas.edu

Office: RLM 9.152

Phone: 471.8131

Office hours: 1:30–3:30 Tuesdays, and by appointment

Homepage: <http://www.ma.utexas.edu/users/danknopf>

Teaching Assistant: Roberta Guadagni
Email: rguadagni@math.utexas.edu

Resources: This course will use BLACKBOARD. Lectures and class announcements will be posted there. We will maintain a discussion board, called *MathChat*, where you may submit questions and share answers. Your TA and I will check these frequently, answering your questions as promptly as possible.

Syllabus: This syllabus may be updated during the semester. A current version will always be available on BLACKBOARD as well as through a link from my homepage (above).

Grading policy

Regular attendance and active participation are vital to success in this course!

Your final grade will be based on the following components:

- Four in-class exams, each worth 15% of your total grade. (See schedule below.)
- A semester project, worth 15% of your grade. You will prepare a brief (5–7 minute) video designed to introduce your peers (e.g. other *Plan-II* students) to a mathematical concept that extends those studied in class. You will not be graded on production values (your video may be prepared using a smart phone or similar device) but rather on how well you motivate and explain your chosen topic. (See schedule below.)
- Five homework assignments, collectively worth 15% of your grade. (See schedule below.) Many assignments will involve writing essays; others may ask you to collect materials (e.g. photographs) illustrating concepts being discussed in class.
- Class participation, worth 10% of your grade. In addition to participating in class discussions and activities, you will complete daily “minute papers” — brief paragraphs written to ask questions about class or respond to questions raised during class. You will receive points for completing these but will not be judged on their content.

There will be no final exam.

Your overall grade will be computed using the following scale:

D-	D	D+	C-	C	C+	B-	B	B+	A-	A
51–55	56–63	64–65	66–67	68–75	76–77	78–79	80–87	88–89	90–91	92–100

Course materials

Doing the readings is an important component of success in this course.

Our main source will be a custom course packet, available from the University Co-op. The packet contains readings are excerpted from the following texts:

- *The Parsimonious Universe: Shape and Form in the Natural World*, by Stefan Hildebrandt and Anthony Tromba.
- *Fearful Symmetry: Is God a geometer?*, by Ian Stewart and Martin Golubitsky.
- *Architecture and Geometry in the Age of the Baroque*, by George L. Hersey.
- *Experiencing Geometry: Euclidean and Non-Euclidean with History*, by David Henderson and Daina Taimiņa.
- *Geometry of Design: Studies in Proportion and Composition*, by Kimberley Elam.
- *Geometry and the Liberal Arts*, by Daniel Pedoe.
- *Connections: The Geometric Bridge between Art and Science*, by Jay Kappraff.
- *On growth and form*, by D’Arcy Wentworth Thompson.

Course schedule

The following is only a broad outline of the course schedule. Exact topics and contents of assigned readings will be announced as the course progresses. The schedule below may be altered for pedagogical reasons. It is your responsibility to be aware of any changes announced in class.

Thursday, August 30:	Introduction.	
Tuesday, September 4:	Optimality and Modeling.	
Thursday, September 6:	Optimality and Modeling.	<i>Assignment 1 due.</i>
Tuesday, September 11:	Optimality and Modeling.	
Thursday, September 13:	Optimality and Modeling.	<i>Assignment 2 due.</i>
Friday, September 14:	<i>Last day to drop for a possible refund.</i>	
Tuesday, September 18:	Symmetry and Transformations.	
Thursday, September 20:	<i>Exam I</i>	
Tuesday, September 25:	Symmetry and Transformations.	
Thursday, September 27:	Symmetry and Transformations.	
Tuesday, October 2:	Symmetry and Transformations.	
Thursday, October 4:	Symmetry and Transformations.	<i>Project topic due.</i>
Tuesday, October 9:	Symmetry and Transformations.	
Thursday, October 11:	Mathematical Epistemology.	<i>Assignment 3 due.</i>
Tuesday, October 16:	<i>Exam II</i>	
Thursday, October 18:	Mathematical Epistemology.	
Tuesday, October 23:	Mathematical Epistemology.	
Thursday, October 25:	Mathematical Epistemology.	<i>Assignment 4 due.</i>
Tuesday, October 30:	Mathematical Epistemology.	
Thursday, November 1:	Pattern and Abstraction.	<i>Project outline due.</i>
Tuesday, November 6:	Pattern and Abstraction.	<i>Last day to register pass/fail.</i>
Thursday, November 8:	<i>Exam III</i>	
Tuesday, November 13:	Pattern and Abstraction.	
Thursday, November 15:	Pattern and Abstraction.	<i>Assignment 5 due.</i>
Tuesday, November 20:	Pattern and Abstraction.	
Thursday, November 22:	<i>Thanksgiving Holiday — no class</i>	
Tuesday, November 27:	Presentation and discussion of class projects.	
Thursday, November 29:	Presentation and discussion of class projects.	
Tuesday, December 4:	Presentation and discussion of class projects.	
Thursday, December 6:	<i>Exam IV</i>	

Policies

Academic integrity: Any academic dishonesty will be severely penalized. Your project and assignments should be your own work. Be very careful to avoid plagiarism! For a tutorial on what constitutes plagiarism, see:

[http : //www.lib.utexas.edu/services/instruction/learningmodules/plagiarism/](http://www.lib.utexas.edu/services/instruction/learningmodules/plagiarism/)

For tips on avoiding plagiarism and general UT policies about it, see:

[http : //www.utexas.edu/cola/centers/cwgs/_files/pdf-4/ai2012.pdf](http://www.utexas.edu/cola/centers/cwgs/_files/pdf-4/ai2012.pdf)

[http : //deanofstudents.utexas.edu/sjs/acadint_plagiarism.php](http://deanofstudents.utexas.edu/sjs/acadint_plagiarism.php)

Also note that no books, notes, computers, or mobile phones are allowed during exams.

Accommodations: Students with disabilities should request appropriate academic accommodations from the Division of Diversity and Community Engagement, Services for Students with Disabilities. Visit <http://www.utexas.edu/diversity/ddce/ssd/> or else call 512.471.6259. Please inform me of any approved accommodations as early in the semester as possible, and no later than one week prior to the first exam.

Religious holidays: By UT Austin policy, you must notify me of your pending absence at least fourteen days prior to the date of observance of a religious holy day. If you must miss a class, an examination, an assignment, or a project in order to observe a religious holy day, you will be given an opportunity to complete the missed work within a reasonable time after the absence.

Safety recommendations: Please note the following guidelines.

- Occupants of buildings on the University of Texas at Austin campus are required to evacuate buildings when a fire alarm is activated. Alarm activation or announcement requires exiting and reassembling outside.
- If you require assistance in evacuation, please inform me of this fact in writing during the first week of class.
- Familiarize yourself with all exit doors of each classroom and building you may occupy. Remember that the nearest exit door may not be the one you used when entering the building.
- In the event of an evacuation, follow the instruction of faculty or instructors.
- Do not re-enter a building unless given instructions by one the following: Austin Fire Department, the University of Texas at Austin Police Department, or a Fire Prevention Services officer.
- If you have concerns about yourself or a classmate, you may contact the *Behavior Concerns Advice Line* at 512.232.5050.
- For further information, see www.utexas.edu/emergency or contact the Office of Campus Safety and Security, <http://www.utexas.edu/safety/>, 512.471.5767.