Exam 1 30 min

Let $x_1 = 1; x_2 = 2$ and for $n \geq 2$, $x_{n+1} = x_n \cdot x_{n-1}$. Prove that for all $n$,

$$x_n \leq 2^{2^n}$$

a) Don't try to simplify $2^{2^n}$. It isn't $4^n$ or $2^{2n}$ or anything like that.
b) This is all about exponents: $2^a \cdot 2^b = 2^{a+b}$

I'll prove by strong induction.

a) Check $P(1)$: $x_1 \leq 2^1 \vee 1 \leq 2^2 \vee 1 \leq 4 \vee$

Check $P(2)$: $x_2 \leq 2^2 \vee 2 \leq 2^4 \vee 2 \leq 16 \vee$

b) Now show $P(1) \land P(2) \land \ldots \land P(k) \rightarrow P(k+1)$ for $k \geq 2$.

Since $k \geq 2$,

$$x_{k+1} = x_k \cdot x_{k-1}$$

$$\leq 2^{2^k} \cdot 2^{2^{k-1}}$$

$$\leq 2^{2^{2k}}$$

$$x_{k+1} \leq 2^{2^{k+1}}$$

$P(k+1)$

So $P(n)$ is true for all $n$.

(Algebra below)

\[
2^{2^k} \cdot 2^{2^{k-1}} = 2^{2^k + 2^{k-1}} = 2^{2^k + 2^{k-1}} = 2^{2^k} + 2^{2^{k-1}} = 2^{2^{k+1}}
\]