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1) Prove that a natural number \( n \) is even if and only if \( n^2 \) is even; a natural number \( n \) is odd if and only if \( n^2 \) is odd.

2) Let \( x_1 = 1; x_2 = 1; x_{n+1} = x_n + x_{n-1} \). Prove that for all natural numbers \( n \),

\[
1 \leq x_n \leq 3^n
\]

3) Assume \( x_1 \) is defined so that \( 0 < x_1 < 1 \) and then define \( x_n \) recursively as

\[
x_{n+1} = x_n(1 - x_n)
\]

Prove that for all \( n \), \( 0 < x_n < 1 \)