1) (30 points) Let $S$ be a bounded non-empty set of real numbers. Assume $u$ is a real and that for all $n \in \mathbb{N}$, $u - \frac{1}{n}$ is not an upper bound of $S$. What can you say about $u$? Prove your answer.

2) (40 points) use the definition of limit ($\epsilon$, $K$ proof) to prove that the following converges:

$$\frac{2n^2}{n^2 - 5n - 6}$$

3) (30 points) $a_n$ be a sequence. we say $a_n \searrow a$ if

$$\forall \epsilon > 0 \forall K \forall n [n \geq K \Rightarrow |a_n - a| < \epsilon]$$

Find a sequence $a_n$ and a number $a$ such that $a_n \to a$ but it is not true that $a_n \searrow a$. 

Old Exam