1) Let $S$ be a standard set, bounded above. Assume $b$ is an upper bound of $S$, and $b \in S$.
Prove $\sup(S) = b$.

2) Let $S = \{x \mid 0 < x < 1\}$. Prove $\inf(S) = 0$.

a) I have to show
   b) $b$ is an upper bound of $S$. This was given
   ii) If $b$ is any upper bound of $S$, $b \leq b_1$.
   But $b \in S \to a \leq b$, definition of upper bound
   so $b \leq b$.

b) I have to show
   i) 0 is a lower bound of $S$.
   $a \in S \to 0 < a < 1 \to 0 < a \to 0 \leq a$. So 0 is a lower bound.

ii) Let $l$ be any lower bound of $S$. I have to show $l \leq 0$.
   Assume $l > 0$; I'll show $l$ is not a lower bound of $S$,
   and thus get a contradiction. So I have to find $a \in S$ with $l < a$.
   Case 1) $l \leq 1$. Then let $a = \frac{1}{2}$, $\frac{1}{2} \in S$ and $\frac{1}{2} < l \leq l$. so thus
   gives the contradiction.
   Case 2) $0 < l < 1$. Let $a = \frac{l}{2}$. Then $0 < a < l$, $0 < a \leq 1$
   or $0 < a < 1$ so $a \in S$.
   Now I'll show $l < a$
   $\Rightarrow \frac{a}{2} < a$
   $\Rightarrow \frac{1}{2} < l$ since $l > 0$
   $\Rightarrow$ true.