Group Work: Limit Laws

True or False? Prove or give a counterexample:

3) If $x_n$ converges and $y_n$ diverges then $x_n + y_n$ diverges.

4) If $1/y_n$ diverges then $y_n \to 0$.

5) If $1/y_n \to 0$ then $y_n \to \infty$.

6) $x_n y_n$ converges and $y_n$ is unbounded then $x_n \to 0$.

7) $x_n$ converges and $y_n \to 0$ then $x_n/y_n$ is unbounded.
Group Work: Limit Laws - Solutions

1) False: Suppose given in class:
\[ a_n = (-1)^n, \quad b_n = (-1)^n. \] Both diverge.
\[ a_n + b_n = (-1)^n + (-1)^n = (-1)^n (1 + (-1)^n) = 0. \] Converges.

2) True: Of \[ x_n + y_n \] and \[ x_n \] converge, then \[ x_n + y_n + (-1) x_n \] also converges -
Limit Law Addition.
\[ But \ this \ is \ y_n. \ So \ y_n \ converges. \]

3) This is true. We’ll do a proof by contradiction.
So assume \[ x_n \] converges \[ y_n \] diverges \[ x_n + y_n \] converges.
By problem 2), \[ y_n \] converges contradicting divergence hypothesis.

4) False. Let \[ y_n = (-1)^n. \] Then \[ \frac{1}{y_n} = (-1)^n. \] Both diverge.

5) False - wrongly given in class -
Let \[ y_n = -n. \] Then \[ \frac{1}{y_n} = -\frac{1}{n} \to 0. \] V.
But \[ y_n \] does not go to infinity; \[ y_n \] is not even positive.
6) False. Let \( x_n = (1, 0, 1, 0, 1, 0, \ldots) \) and \( y_n = (0, 1, 0, 2, 0, 3, \ldots) \). Then \( y_n \) is unbounded, \( x_n y_n = 0 \) converges to 0, but \( x_n \) does not converge at all.

7) False. Let \( x_n = y_n = \frac{1}{n} \). Then \( y_n \to 0 \). So \( x_n = y_n \) converges to 0. But \( x_n / y_n = 1 \). Which is not unbounded.