Work Group Problem 2

Quiz 1  Time: 20 min

1) Use induction to prove that if \( s_n = 1 + 2 + 3 + \cdots + n \), then \( s_n \geq 2n^2 \)

a) Find a recursion formulas for \( s_n \)
b) Find a predicate \( P(n) \) for your induction

Now do the proof.

\[ s_1 = 1 \quad s_2 = 1 + 2 = s_1 + 2 \quad s_3 = 1 + 2 + 3 = s_2 + 3 \]

Solution: \( s_1 = 1 \) \quad \( s_{n+1} = s_n + (n+1) \)

b) \( P(n) \) & \( s_n \leq 2n^2 \).

c) Prove for all \( n \), \( P(n) \) is true, by induction.

i) Base \( P(1) \): \( s_1 \leq 2 \cdot 1^2 \) definition of \( s_1 \) \( 1 = 2 \) definition of \( s_1 \)

ii) Prove \( P(k) \rightarrow P(k+1) \)

\[ s_{k+1} = s_k + (k+1) \] definition of \( s_{k+1} \)
\[ s_k + (k+1) \leq 2k^2 + (k+1) \text{ induction hypothesis} \]
\[ 2k^2 + (k+1) \leq 2(k+1)^2 \quad \text{pf below} \]
\[ 2k^2 + (k+1) \leq 2(kk+1)^2 \text{ transitivity if} \]
\[ s_{k+1} \leq 2(k+1)^2 \text{ so, for all } n, P(n) \text{ is true} \]

pf below
\[ 2k^2 + (k+1) \leq 2(kk+1)^2 \Rightarrow \]
\[ 2k^2 + (k+1) \leq 2(k^2 + 2k + 1) \Rightarrow \]
\[ 2k^2 + (k+1) \leq 2k^2 + 4k + 2 \Rightarrow \]
\[ 0 \leq 3k + 1 \]

true: \( k \) is the terms on \( -k \) is positive.