Q5 Sample

1) Let \((x_n), (y_n)\) be sequences. Assume for all \(\varepsilon > 0\) there exists a \(K_0\) such that \(n \geq K_0\) implies \(|x_n - y_n| < \varepsilon\). Assume \(y_n \to 0\). Use \(\varepsilon, K\) techniques to prove \(x_n \to 0\).

Given \(\varepsilon > 0\), \(\varepsilon/2 > 0\) as well, so there exists a \(K_1\) with \(n \geq K_1 \Rightarrow |y_n - 0| < \varepsilon/2\). Let \(K = \sup \{K_1, K_2\}\).

\(n \geq K_2 \Rightarrow |x_n - y_n| < \varepsilon/2\) or \(|x_n - 0| \leq |x_n - y_n| + |y_n - 0|\).

If \(n \geq K_2\) so \(|x_n - y_n| < \varepsilon/2\).

And \(n \geq K \Rightarrow n \geq K_1\) so \(|y_n - 0| < \varepsilon/2\) so

For \(n \geq K\), \(|x_n - 0| < \varepsilon/2 + \varepsilon/2 = \varepsilon\).

The idea here is that we know about \(y_n\), and we know about \(x_n - y_n\), so we have to express \(x_n\) in terms of these two. It isn't hard: \(x_n = x_n - y_n + y_n\), so

\(|x_n| = |x_n - y_n + y_n|\). Now the limit is need

\(x_n - y_n\) and \(y_n\) close, so

\(|x_n - y_n + y_n| \leq |x_n - y_n| + |y_n| < \varepsilon\)

Now let each of \(|x_n - y_n|\) and \(|y_n|\) have

half of \(\varepsilon\), and we get my \(\varepsilon/2\) proof.